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HULL Users' Manual

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This report outlines the structure and input syntax for using the HULL finite difference hydrodynamics code. HULL solves the partial differential equations descriptive of an inviscid, non-conducting continuous hydrodynamic/elastic/plastic medium subjected to dynamic loading.

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PREFACE

This report was prepared by Orlando Technology, Incorporated, P. O. Box 855, Shalimar, Florida 32579, under Contract No. F08635-81-C-0131 with the Air Force Armament Laboratory, Armament Division, Eglin Air Force Base, Florida 32542. Mr. William Cook (DLJW) managed the program for the Armament Laboratory. The work was accomplished during the period from September 1982 to September 1983.

The Public Affairs Office has reviewed this report, and it is releasable to the National Technical Information Sservice (NTIS), where it will be available to the general public, including foreign nationals.

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

Milton D. Kingcaid

MILTON D. KINGCAID, Colonel, USAF
Chief, Munitions Division

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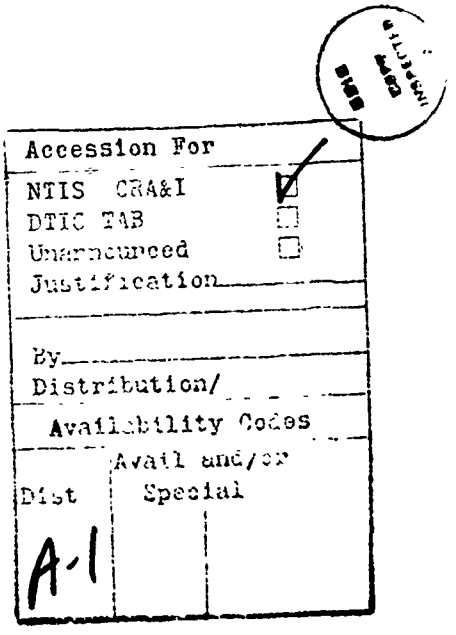

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TABLE OF CONTENTS

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LIST OF FIGURES

LIST OF TABLES

SECTION I
INTRODUCTION

HULL is a system of computer programs which solve the finite-difference analogs of the partial differential equations descriptive of a non-conducting, inviscid, elastic/plastic continuous medium subjected to dynamic loading.

HULL is capable of running cylindrical or cartesian coordinate calculations in two spatial dimensions or cartesian coordinate calculations in three dimensions. HULL is a multimaterial Eulerian code which employs a diffusion limiter to preseve material interfaces. Up to ten materials can be handled in a single calculation. A mixture of any of these materials is permitted in any one cell. Solid material properties are modeled by the Mie-Gruneisen equation of state (Reference 1). Plasticity is modeled by the Von Mises yield criteria which can be modified by plastic work hardening and thermal softening. Slip surfaces are automatically handled at material and failure interfaces. Failure can be determined by simple stress, strain, or tri-axial criteria. Failure is modeled by inserting a void or low pressure gas in the failed zone and allowing the tensile stresses to relax. Recompression of the failed zone is not inhibited.

Large computational meshes can be handled by HULL since only a small portion of the mesh need be in central memory as the calculation proceeds. Time snap-shots at arbitrary intervals can be retained on permanent disk or tape files for subsequent analysis or problem restart. The HULL system has several service routines for keeping track of this data storage and for providing graphical presentation of most features of interest.

Provisions are available to link HULL to the Lagrangian TOODY code (Reference 2) or EPIC3 code (Reference 3) for calculations in two and three dimensions, respectively.

Development of HULL (Reference 4) started in 1971 at the Air Force Weapons Laboratory (AFWL). This initial version was built to solve nuclear weapons effects problems with special emphasis on nuclear fireball rise and air blast propagation. In 1975 a conventional weapons effects version of HULL

(Reference 5) was developed at the Air Force Armament Laboratory (AFATL). This later version of the code incorporated new physics modules for calculating hydrodynamic and elastic/plastic forces. In addition, the code architecture was modified to permit vectorization on pipe-line machines such as the CRAY. HULL has been continuously developed by the AFATL and AFWL since 1975 with primary management by AFATL. Previous versions or offshoots of HULL are not currently supported by the AFATL/AFWL team.

SECTION II
THE HULL SYSTEM

The HULL system of codes is composed of:

(1) KEEL - an initial condition (or mesh) generator,

(2) HULL - the finite difference code,

(3) PULL - the graphics package,

(4) BOW - the dump tape library,

(5) PLANK - the alternate input generator for SAIL

(6) CONTROL - an interactive program status monitor, and

(7) SAIL - the system file manager and executive processor.

All programs or codes in the HULL system are maintained on a single SAIL program library file. Compilation of any program element of HULL entails execution of SAIL to process the program library, select the proper subroutines and options desired for the particular application, and produce a file which will serve as input to the compiler. This processing activity is done to keep storage requirements and central processor time requirements to the practical minimum consistent with code comprehension and structure flexibility. The use of SAIL for running HULL and its associated service programs will be addressed in this document. Other documents which describe the HULL code are limited to specific features or elements of HULL (References 6, 7, and 8). Use of SAIL for program alteration and file maintenance is covered by Reference 8.

Figure 1 illustrates the interactions of the program elements of the HULL system. Each of the programs are described more completely in the respective sections of this report.

The general setup and running of a HULL calculation (assuming that an executable SAIL module and the HULL system file are available on the computer system) starts with the formulation of an input deck for KEEL. This input deck contains all the specifications for formulating the initial conditions for the calculation. The KEEL input is first read by PLANK. PLANK processes

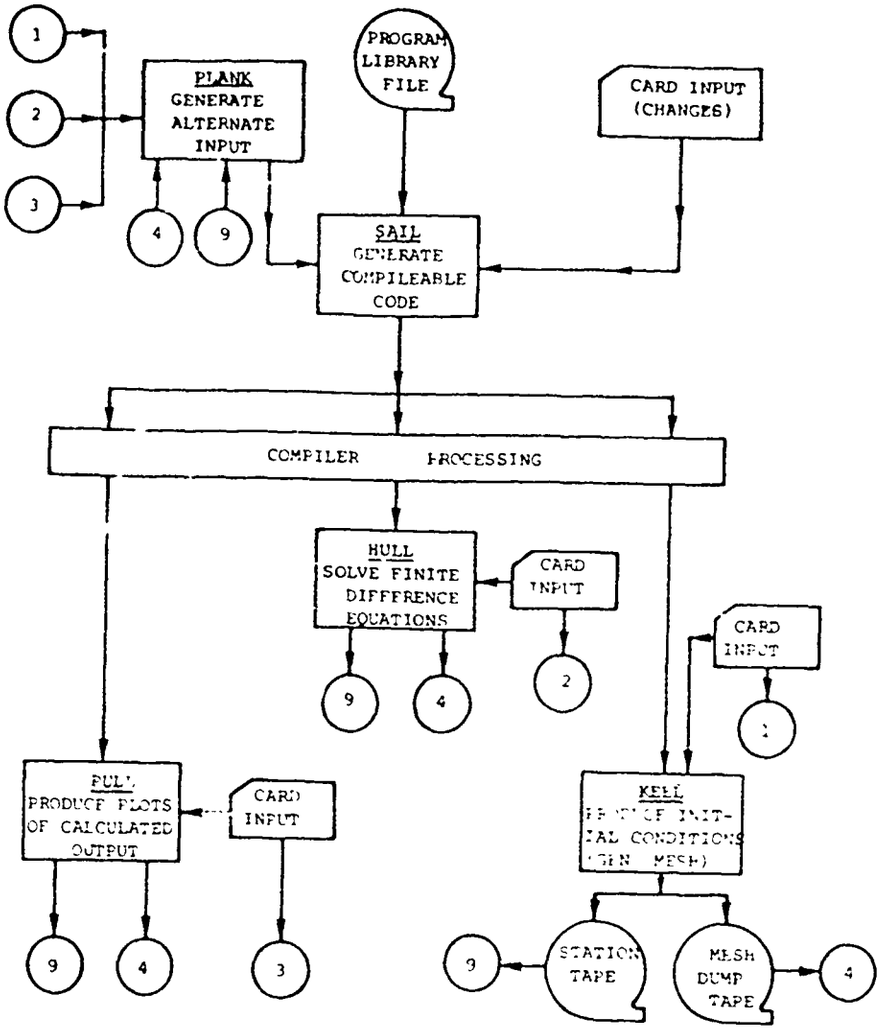

Figure 1. HULL System Data Flow Schematic

the deck to determine the mesh size and options to be used for this run. PLANK then generates a disk input file for SAIL which details the structure of program KEEL. SAIL uses this disk file to process the HULL system library file and assemble the KEEL compiler input. The compiler input is compiled and the resulting code is loaded and executed as program KEEL. KEEL reads the same input deck used for the generation of KEEL. KEEL then produces the HULL problem initial conditions and puts the results on logical unit 4. Unit 4 is either a physical tape or a permanent file disk device which will store the data for running HULL (or PULL). A second optional tape file output is produced if detailed time history data at several arbitrary HULL mesh points are desired. This is called the station tape, and it resides on logical unit 9. The station tape is required for linking HULL to the Lagrangian TOODY or EPIC3 codes.

The output data from KEEL on unit 4 is used as input to PLANK to help SAIL tailor the HULL program for a particular run. This tailoring process can be modified by the HULL input deck as will be described later. PLANK and SAIL produce the compiler input. When HULL is compiled and loaded, it reads the initial conditions from unit 4 and starts the calculation. Snap-shots of the proceeding calculation at pre-determined times (from input) are put on unit 4 for later analysis or restart. Station data is also accumulated and recorded on unit 9 if it is present. Unit 4 snap-shots or dumps are also made pericdically every 60 central processor minutes to protect against unscheduled machine failures. Thus, unit 4 and unit 9 store a continuous record of the running calculation that is sufficient to restart the calculation at any of the available unit 4 dump times.

Unit 4 and unit 9 are the permanent record of the calculation for subsequent analysis and editing. PLANK uses these tape files and a PULL input deck to instruct SAIL in the generation of the PULL code. PULL produces an exhaustive set of graphical outputs which can be selected for problem analysis. The PULL code, after being generated by SAIL and compiled by the operating system, reads unit 4 or unit 9 and produces graphics output for the selected plotting devices.

SECTION III
PROGRAM SELECTION AND CONTROL

HULL System elements are selected and configured by the SAIL code with instructions from the input data stream and PLANK. The default option directory on the SAIL library file is used to simplify this procedure by remembering the primary option set values. The HULL system default options are all keyed from the installation parameter INST and have the value and installation definition in the following table.

Value of INST	Installation Definition
1	Air Force Weapons Laboratory (Kirtland AFB, NM)
2	Air Force Armament Laboratory (Eglin AFB, FL)
3	McDonnel Douglas Astronautics (Huntington Bch, CA)
4	Army Ballistic Missile Defense Agency (Huntsville, AL)
5	Science Applications Incorporated (Huntsville, AL)
6	Royal Armament Research and Development Establishment (Fort Halstead, Kent, UK)
7	Army Ballistic Research Laboratory (Aberdeen, MD)
8	Vought (Dallas, TX)
9	OTI (Shalimar, FL)

Once the installation parameter is set, the options which choose the computer system specific parameters are selected automatically.

After the correct options have been assembled to tailor the HULL programs for the installation machine and operating system, we can further control SAIL processing to produce the specific form of KEEL, HULL, or PULL needed to run a given problem. The dump tape (unit 4) produced by KEEL is the major control for setting up programs HULL and PULL. Most of the input to KEEL sets these control parameters through a unit 4 record called the ZBLOCK. The names of the ZBLOCK parameters, the meaning of their various values, and the defaults are given below.

ZBLOCK Name (Alternate name)	Values (Default)	Definition
PROBLEM (PROB)	Any number of the form XXXX.YYYY	Arbitrary problem identifier
AREF	True., (False.)	Y(j=1) boundary reflective (3D)
ATMOS	Integer 1	tropical atmosphere
	2	temperate atmosphere
	3	arctic atmosphere
	4	exponential atmosphere
	(5)	constant pressure and density
BREF	.TRUE., (.FALSE.)	bottom boundary reflective Y(j=1), 2D; Z(k=1), 3D
BURN	Integer (0)	no burn routine
	1	burn front advanced by fluid state
	2	programmed burn
CODE	Integer (1)	normal hydrodynamic - elastic/ plastic formulation
	2	include interactive dust
COLD	(.TRUE.), .FALSE.	radiation cooling if false
CYCLE	Real	number of time steps from initial conditions
DIMEN	(2), 3	number of spatial dimensions
DT	Real	time step (seconds)
ELC	Real	energy last check, current total mesh energy

EOS	1	Doan-Nickel air equation of state
	2	Constant gamma equation of state
	(6)	Multi-material Mie-Gruneisen equation of state
	10	tabulated air equation of state
ETH	Real	theoretical total mesh energy
EXPAND	Real number <1(.10)	mesh expansion fraction for rezone
FAIL	Integer (0)	no failure model
	1	simple stress criteria (STRAIN=0)
		simple stress and strain criteria
		(STRAIN = 1)
FLUXER	Integer 0	no fluxing
	1	single material fluxing
	(3)	volume fluxing with species mass, volume, and energy retained
FREF	.TRUE., (.FALSE.)	Y(j=JMAX) boundary reflective (3D)
GEOM	1	cartesian coordinates
	(2)	cylindrical coordinates (2D only)
HOB	Real (0)	weapon burst height in cm
IMAX	Real (100)	number of zones in x coordinate direction
IQ	Real (IMAX-1)	number of active x coordinate zones
ISLAND	(0)	no effect
	1	internal mesh reflective boundary
	2	reflective boundary with rigid body translation (not implemented in 3D)

JMAX	Real (200)	number of zones in y coordinate direction
JQ	Real (JMAX-1)	number of active y coordinate zones
KMAX	Real (1)	number of zones in z coordinate direction
KQ	Real (KMAX-1)	number of active z coordinate zones (3D)
LREF	(.TRUE.),.FALSE.	X(i=IMAX) boundary reflective
MAGFLD	(0)	no magnetic field
	1	dipole field and MHD formulation
METHOD	0	no calculation in Lagrange step; used for testing fluxer
	(2)	(second order in time)
MLC	Real	mass last check, current total mass in the mesh
NH	Integer	number of variables per zone
	(5 + 3* NM + NHIST; 2D)	calculated by KEEL
	(6 + 3* NM + NHIST; 3D)	
NHIC	Integer (set by PLANK)	number of words in core to retain mesh variables
NHIST	Integer (set by PLANK)	number of fluxed material dependent histories per cell (stress components, plastic work, etc.)
NM	Integer (1)	number of materials
NVARST	Integer (15, 2D; 22, 3D)	number of variables per station
NOP	Integer (0)	number of trace particles and stations
NPLPB	Integer (calculated by PLANK)	number of planes per record block on TAPE4 (3D)

NPP	Integer (2, 2D; 3, 3-D)	number of parameters per particle (usually coordinates)
NROWPB	Integer (calculated by PLANK)	number of rows per record block on unit 4
NSTN	Integer (0)	number of stations
PTSTOP	Real (1.E20)	problem stop time
RAD	Integer (0)	no radiation routines
	1	equilibrium radiation diffusion
	2	non-equilibrium radiation diffusion
RADLOSS	Real	total energy radiated
REZONE	(0)	no rezone
	1	horizontal and vertical - shock triggered
	2	particle triggered - horizontal and vertical
	3	horizontal shock follower
	4	vertical shock follower
	5	vertical particle follower
	6	same as 5 with shocks diffused
	7	continuous (only rezone for 3-D)
	8	vertical material 2 follower
RREF	.TRUE., (.FALSE.)	X(i=IMAX) boundary reflective
STRAIN	Integer (0)	no strain components saved
	1	strains saved (only if STRESS >0)
STRESS	0	no stress (elastic/plastic formulation not included)
	(1)	stress calculation included
SUME	Real	total energy lost by cooling

TERAD	Real	total energy lost by radiation
TLC	Real	time of last check on energy and mass
TREF	.TRUE., (.FALSE.)	Y(j=JMAX), 2D; Z(k=KMAX), 3D boundary reflective
TTIME	Real	total problem central processor time (seconds)
TTSTOP	Real (100.,	total central processor time stop (hours)
UREZ	Real (10.)	x coordinate velocity rezone trigger (cm/sec)
VISC	Integer (0)	no explicit artificial viscosity
	1	explicit artificial viscosity included
VREZ	Real (10.)	y coordinate velocity rezone trigger (cm/sec)
VOIDS	Integer (0)	no voids allowed
	1	implicit voids permitted
	2	explicit voids whenever material VOID is found
WORK	Integer (0)	no plastic work hardening
	1	plastic work hardening included
WREZ	Real (10.)	z coordinate velocity rezone trigger (3D) (cm/sec)
XOB	Real (0.)	x coordinate of burst (3D) (centimeters)
YOB	Real (0.)	y coordinate of burst (3D) (centimeters)
YGND	Real (-1.E20)	location of ground for rezone down control; BREF is set to zero when mesh bottom reaches this value

The ZBLOCK variables are identified on TAPE4 by their names in alphabetic form along with their corresponding values. KEEL will default ZBLOCK variables to the values indicated above. If a different value is desired, it can be input through the KEEL or HULL input data stream.

SECTION IV
PROGRAM KEEL

Program KEEL defines the initial ZBLOCK, the mesh coordinates, and the material property specifications (P, u, v, w, I, M, V) on a cell by cell basis for the entire mesh. Input to KEEL is free format with variable names and values delimited by blank, comma, and equal (=). The KEEL input must start with the word KEEL. This allows program KEEL to search for and find its data independent of its location in the input file. The KEEL input is logically divided into three parts: initial ZBLOCK definition, mesh generation, and material generation. The logic flow of KEEL is indicated by Figure 2.

A. ZBLOCK DEFINITION

The ZBLOCK is literally defaulted for every variable name. The only variables that must be set are to define the number of materials and their names. It is usual to associate a problem number to identify the type of problem being run, a user numeric identifier, and to use the numerals to the right of the decimal point to indicate the sequence or series of the calculation when running parameter studies. If the BOW program library is being used to keep track of problems and their associated tapes, it is imperative that all problem numbers be unique. HULL problems can be assigned problem numbers from 00000.0001 to 99999.9999.

After the problem number is defined, it is necessary to determine the number of zones to be used in each coordinate direction. The HULL code is not sensitive to moderate variations in cell size from zone-to-zone in any given coordinate direction (10 percent). However, it is not tolerant of large aspect ratios (greater than 2.5-3). The number of zones should be chosen to provide enough resolution to properly define material interfaces. Choosing the best number of zones in each coordinate is always a compromise most heavily weighted by economic considerations since the computer time required to run a calculation is inversely proportional to the size of the smallest zone and directly proportional to the total number of zones in the calculation. Therefore, decreasing the zone size by a factor of 2 in a 3D calcula-

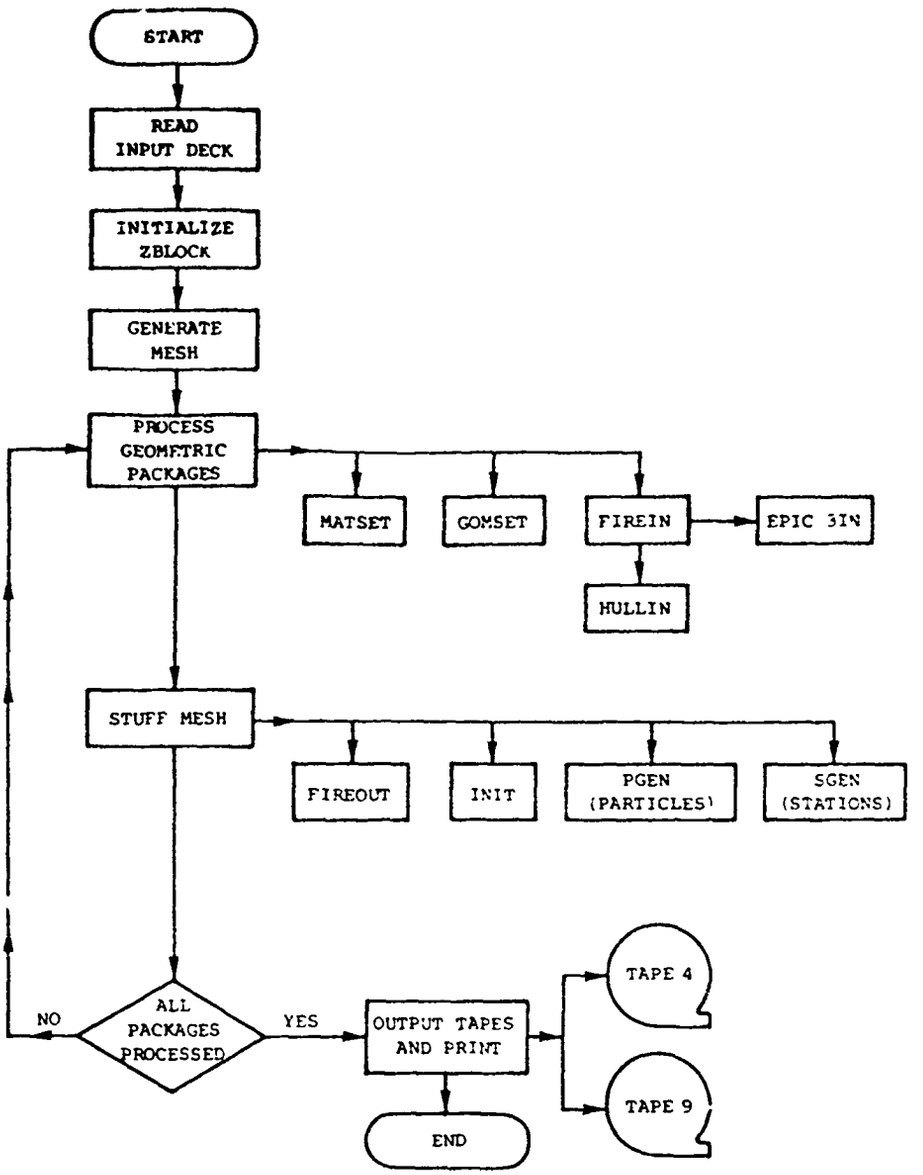

Figure 2. Logic Flow in Program KEEL

tion would result in a factor of 16 increase in the run time. The number of zones in the x, y, and z coordinate directions is set by IMAX, JMAX, and KMAX respectively. Two-dimensional calculations will not use KMAX (the z coordinate).

The next order of business is to select the number of materials and their names. Table B-1 (Appendix B) lists the materials currently recognized by the HULL system. Any material listed in Table B-1 may be input to KEEL, but present code structure limits their total number in a given calculation to ten (i.e., NM$\leq$10). Each material defined must be assigned a number from 1 to NM. The order is arbitrary.

If deviatoric stresses are not to be included, set STRESS = 0. Otherwise, the calculation will assume all materials have strength. If it is desired also to retain the strain components for each cell, set STRAIN = 1. This is necessary if STRAIN or P/Y criteria are to be used for failure. Similarly, if material failure is to be explicitly modelled, it is necessary to set FAIL = 1. This has effect only for STRESS = 1. If FAIL = 1, STRESS = 1, and STRAIN = 0; failure will be determined by a default stress criteria; maximum principal $\geq$ 2.5 yield stress. If STRAIN = 1, both the stress criteria and a principal strain $\geq$ .7 will initiate failure. These are default values and should be used with caution because of our current poor understanding of failure initiation.

Rezone choice may be made by indicating the number from 0 to 7 for the particular rezone selected. REZONE = 0 implies no rezone. This can be altered later as HULL is running. Only REZONE = 7 is supported in three dimensions. This rezone (in 2D or 3D) can, in general, cause a continuous translation of the mesh coordinates in any direction. It is currently implemented to translate the entire mesh at a continuous velocity in the vertical direction (y direction in 2D; z direction in 3D). The translation velocity is set by the zone which contains station or particle number one. Use of the continuous rezone may reduce the numerical method's implicit viscosity to an unacceptable level. In any case it is best to run most dynamic solid calculations with explicit artificial viscosity by setting VISC = 1.

If it is desired to run with Lagrangian tracer particles, the variable NOP should be set to a value greater than the number of particles to be generated. The particle routine will count and set NOP to the actual number inserted. If stations are to be generated, NSTN should be set to a number greater than the number to be inserted. Since all stations are also particles, NOP must be greater than the number of particles plus the number of stations. If a physical tape is to be used to store station data (and neither BOW nor the permanent files are being used), then it is necessary to set STAPE = VSN, where VSN is the volume serial number of the tape. This will only function on CDC machinery and results in the automatic issue of the tape request by KEEL. Otherwise, you must specify the station tape unit 9 by means of control cards.

The stability factor STABF (fraction of the Courant condition time step) is currently defaulted to .75. The default cycle one time step is 1×10^{-7} seconds. If the problem type being run has had difficulties starting during previous HULL runs, DT should be reduced.

For most circumstances an initial time step DT of 10^{-10} seconds should suffice. If STABF has to be set to much less than 0.5, something is seriously wrong; check your initial conditions.

With the current version of HULL (Version 117), ISLAND = 2 and REZONE = 1, 2, 3, 4, 5, 6 have not been implemented in the three-dimensional HULL code.

If the problem being generated has a mesh region that is inactive (no velocity, high pressures, or explosive burn region), some initial problem computer time can be saved by setting IQ, JQ, and KQ (3D). This will cause the mesh in that part of the coordinate beyond X(IQ), Y(JQ), or Z(KQ) to be left out of the calculation until the activity is such that the velocity in any of the active region limit exceeds UREZ, VREZ, or WREZ for the X, Y, and Z coordinate directions, respectively. This feature can not be employed in conjunction with the continuous rezone (if REZONE = 7). As the activity reaches the limits of the active region, the corresponding activity index IQ, JQ, or KQ will be incremented one zone per cycle until it reaches the mesh

limit IMAX-1, JMAX-1, or KMAX-1. The defaults for IQ, JQ, and KQ are IMAX-1, JMAX-1, and KMAX-1.

The last input in this section of KEEL is the header title. This is 80 columns of whatever information the user wishes to use to identify the calculation. This title will be printed at the beginning of the output for each HULL run and will appear in the titles of all plots produced by PULL. The 80 columns of information must be preceded by a card image with the word HEADER.

The end of ZBLOCK data definition is signalled by the word MESH.

An example of this part of the KEEL input for a three-material problem to calculate the stress induced in an RHA plate impacted by copper and ignoring failure could be set up by:

```
KEEL PROB=1001.001
NM=3  AIR=1  RHA=2  CU=3  DIMEN=2
STRESS=1  VISC=1  DT=1.E10
REZONE=0  IMAX=80  JMAX=120
HEADER
TEST CALCULATION TO ILLUSTRATE KEEL INPUT
MESH
```

If it is desired to insert stations to record stress time histories at 10 points, then

```
NOP=11, NSTN=11
```

could be placed between or on any card preceding HEADER. The almost explicit (with aid of the ZBLOCK) definition makes this portion of KEEL input relatively easy to use.

B. MESH GENERATION

The next logical element of the KEEL input is definition of the mesh coordinates. This part of the KEEL input begins with the keyword MESH. Several options are possible. The simplest is a linear mesh in all coordi-

nate directions. This is set up by establishing the mesh boundaries in each coordinate direction. The variable names are:

X0	coordinate of X(I=0)
Y0	coordinate of Y(J=0)
Z0	coordinate of Z(K=0) (3D only)
XMAX	coordinate of X(I=IMAX)
YMAX	coordinate of Y(J=JMAX)
ZMAX	coordinate of Z(K=KMAX) (3D only)

The mesh coordinates are established for the boundary of each zone having indices (I, J, K) at the position midway between its center and zones (I+1, J, K), (I, J+1, K), and (I, J, K+1). Thus, the definition of X0, Y0, and Z0 is to locate left, back, and bottom of the first cell. Each of these variables must be paired with a numeric value. There are restrictions. In two dimensions only X0=0 is allowed. In all cases X0, Y0, and Z0 are defaulted to zero. The zones will be of constant size for this mesh specification with zone size:

```
DX=(XMAX-X0)/IMAX
DY=(YMAX-Y0)/JMAX
DZ=(ZMAX-Z0)/ZMAX (3D only)
```

Thus, for the values of IMAX=80 and JMAX=120 in the preceeding sections, the input

```
MESH XMAX=40  Y0=30  YMAX=90
```

would produce a mesh with DX=DY=0.5 centimeter.

There are usually economic constraints to running calculations with zones small enough for sufficient resolution of all effects everywhere in the mesh. The best compromise frequently is to concentrate a fine constant grid in the region of most interest and increase the zones in all directions away from that region. This is especially important for reasonable treatment of solids since perfectly transmissive mesh boundaries for the HULL code have not yet been formulated. Thus, if the calculational mesh is too small, disturbances will partially reflect from its boundary and propagate to the interior of the

mesh. These reflected disturbances may disrupt the validity of the calculation in the region of interest. This mesh generation option is obtained with the keywords:

```
MESH
CONSTANT SUBGRID NX=NN  XO=X  XMAX=XX
                 NY=MM  YO=Y  YMAX=YY
                 NZ=LL  ZO=Z  ZMAX=ZZ (3D)
```

NX, NY, and NZ are the number of zones in the constant subgrid region with boundaries XO, XMAX; YO, YMAX; and ZO, ZMAX respectively. The zone dimensions within this region are defined by

```
DX=(XMAX-XO)/NX
DY=(YMAX-YO)/NY
DZ=(ZMAX-ZO)/NZ (3D).
```

NX, NY, and NZ are defaulted to IMAX, JMAX, and KMAX, respectively. The remaining IMAX-NX, JMAX-NY, and KMAX-NZ zones outside the constant region will be increased in size by a constant factor as the zone index increases beyond NX, NY, or NZ. The factors are RXPOS, RXNEG; RYPOS, RYNEG; and RZPOS, RZNEG for zones with indices greater or less than the constant subgrid boundary in the X, Y, and Z coordinate directions, respectively. The default value is 1.1 for all these factors, i.e., a 10-percent growth rate will be applied for adjoining zones as they get progressively further from the constant subgrid region. This is a good value to use in most situations. If it is desired to have the exterior limits of the mesh stop near some limit, then the variables:

```
XOLIM=X  XMLIM=XX
YOLIM=Y  YMLIM=YY
ZOLIM=Z  ZMLIM=ZZ
```

can be specified along with a large value of the corresponding multiplying factors. The result in the X coordinate direction would be to place $X(0) \geq$ XOLIM and $X(IMAX) \leq$ XMLIM. The other coordinates would be treated in a similar manner for their corresponding variables.

If the above lacks flexibility for a particular application, the user can specify the size of each zone with the variable name/value pairs

```
MESH NX=i1  DX=x1  NX=i2  DX = x2
     NY=j1  DY=y1  NY=j2  DY = y2
     NZ=k1  DZ=z1  NZ=k2  DZ = z2
```

where i1, i2 are the number of zones with zone size x1 for the first i1 zones, and zone size x2 for the next i2 zones until the sequence sum is equal to IMAX. X0 defines the origin. The mesh coordinates Y and Z will be handled in an analogous way. The individual value of each occurrence of NX, NY, and NZ can be from one to IMAX, JMAX, and KMAX, respectively. Thus, for a brute force mesh setup one could specify

```
MESH Y0=0  X0=0

     NX=1  DX=.1  NX=2  DX=.2  NX=3  DX=.3

     NY=2  DY=.1  NY=4  DY=.5  NY=2  DY=1
```

would produce mesh coordinates (assuming IMAX=6 and JMAX=8) of:

X_i=0, .1, .3, .5, .8, 1.1, 1.4; i=integers [0, 6]

Y_j=0, .1, .2, .7, 1.2, 1.7, 2.2, 3.2, 4.2; j=integers [0, 8].

The search for additional mesh data will terminate with the keyword GENERATE.

C. MATERIAL GENERATION

The KEEL input data starting with the word GENERATE is used for inserting materials, particles, and stations in their desired mesh coordinates. Although the nuclear options for isothermal spheres and other constructs still remain in the code, they have not been used in several years in this version of the code and probably do not execute properly. Their use is covered in previous documents (Reference 4) and will not be repeated here. Their functions can be replicated by the syntax which follows.

Material can be inserted into the mesh by KEEL from two basic sources--from a previous HULL or EPIC3 calculation, or from the KEEL input data stream. To input data from a previous calculation the keyword FIREIN must be used. This is immediately followed by the word HULL or EPIC3 depending on the data source. The next variable name/value pair to occur is

TAPE=VSN or PFN=permanent file name
ID=permanent file identifier

where VSN is the volume serial number of the tape which contains the HULL data and PFN and ID are the permanent file name and file identifier (ID), respectively, of the EPIC3 data. HULL data is supplied by any previously run HULL calculation in a standard TAPE4 format. EPIC3 data is provided by a special dump routine which is described in Reference 30. The KEEL input at this point would appear as

```
GENERATE
FIREIN   HULL   TAPE=VSN
```

or

```
FIREIN   EPIC3   PFN=pfn   ID=id.
```

The construct for defining the material and location destination in the mesh is:

```
PACKAGE material geom DELETE geom 1 geom 2
```

where material is the HULL name of one of the materials defined in the first part of the KEEL input data stream. This material must also have been present in the calculation which produced the HULL or EPIC3 data tape being input or this package will have no effect. Geom is the geometric figure and its specifications which define the region in the mesh that is to be filled from the input data tape. The geometric figures which follow (geom 1, geom 2 to a total of 9) are deleted from the first geometric configuration. The word DELETE is not necessary, since the appearance of a second and subsequent geometry specification within a PACKAGE implies deletion of that region encompassed by its boundaries.

Each of the geometric figures is accompanied by a list of parameters which define their descriptions and position in the mesh. The following constructs are for describing two-dimensional material generation.

RECTANGLE X1=x1 X2=x2 Y1=y1 Y2=y2

A rectangle with vertex coordinates (x1, y1), (x1, y2), (x2, y1), (x2, y2).

TRIANGLE X1=x1 X2=x2 X3=x3
Y1=y1 Y2=y2 Y3=y3

A triangle with vertex coordinates (x1,y1), (x2,y2), (x3,y3).

CIRCLE XC=x1 YC=y1 RADIUS=r

A circle of radius r with center at (x1,y1).

ELLIPSE A=y1 B=b C=x1 D=d

An ellipse with center (x1,y1) and semiaxis length b in the y coordinate and d in the x coordinate

PARABOLA A=a B=b C=c

All (x_i,y_i); $y_i \leq a+b\ (x_i-c)^2$.

HYPERBOLA A=a B=b C=c D=d

All (x_i,y_i); $y_i \geq a+b\ 1+(x_i-c)^2/d^2$

GNRLFIT A=a B=b C=c D=d E=e F=f

All (x_i,y_i); $y_i \geq ax_i^5 + bx_i^4 + cx_i^3 + dx_i^2 + ex_i + f$.

CURVE TABLE NPT = n (x1,y1) (x2,y2)---(xn,yn)

For $n \leq 100$ points and $x1 < x2 < x3 < \text{---} < xn$

All (x_i,y_i); $x_m \geq x_i \geq x_{m+1}$ and $y_i \geq y_m + (x_i-x_m)\left(\frac{y_{m+1}-y_m}{x_{m+1}-x_m}\right)$

The three-dimensional geometry definitions are described in a similar way.

Box X1=x1 X2=x2 Y1=y1 Y2=y2 Z1=z1 Z2=z2

The inside of the prism with corners (x1,y1,z1), (x1,y1,z2), (x1,y2,z1), (x1,y2,z2), (x2,y1,z1), (x2,y1,z2), (x2,y2,z1), (x2,y2,z2).

WEDGE X1=x1 Y1=y1 X2=x2 Y2=y2 Y3=y3 X3=x3 Z1=z1 Z2=z2

The inside of the triangular prism with base in the XY plane and corners (x1,y1), (x2,y2), (x3,y3) in the planes Z=z1 and Z=z2.

PYRAMID X1=x1 Y1=y1 Z1=z1 X2=x2 Y2=y2 Z2=z2
X3=x3 Y3=y3 Z3=z3 X4=x4 Y4=y4 Z4=z4

The inside of the tetrahedron defined by the triples (x_i,y_i,z_i); i=1,2,3,4.

CYLINDER XC=x1 YC=y1 ZB=z1 ZT=z2 RADIUS=r

The interior of the right circular cylinder of radius r with axis parallel to the Z axis and centered in the XY plane at (x1,y1). The cylinder ends are at Z=z1 and Z=z2.

ELLIPCYL A=a B=b C=c D=d ZB=z1 ZT=z2

The interior of the elliptical cylinder described by the parameters a, b, c, d as in the two-dimensional ellipse with ends at Z=z1, Z=z2.

PARCYL A=a B=b C=c ZB=z1 ZT=z2

The interior of the parabolic cylinder analogous to the ellipse above.

HYPERCYL A=a B=b C=c D=d ZB=z1 ZT=z2

The interior of the hyperbolic cylinder analogous to the ellipse above.

SPHERE XC=x1 YC=y1 ZC=z1 RADIUS=r

The interior of the sphere of radius r at (x1,y1,z1).

RECTAROT X1=x1 X2=x2 Z1=z1 Z2=z2

The region swept by the rectangle specified rotated about the z axis.

TRIROT X1=x1 Z1=z1 X2=x2 Z2=z2 X3=x3 Z3=z3

The region swept by the specified triangle rotated about the Z axis.

CIRCLEROT XC=x1 ZC=z1 RADIUS=r

The region swept by the specified circle rotated about the Z axis.

ELLIPSEROT AZ=a BZ=b CZ=c DZ=d

The region swept by the specified ellipse rotated about the Z axis.

PARABLROT AZ=a BZ=b CZ=c

The region swept by the specified parabola rotated about the Z axis.

GNRLFIT A=a B=b C=c D=d E=e F=f

The region swept by the polynomial analogous to the two-dimensional general fit rotated about the Z axis.

CURVE TABLE NPT=N x1, z1, x2, z2---xn, zn

For $n \leq 100$, the region swept by the analog of the two-dimensional curve table rotated about the Z axis.

All of these geometric figure specification parameter names have synonyms. The following names in each set are equivalent:

SET 1 X0, X1, XL, XLEFT, XC, XCENT, XCNTR, XCENTER

SET 2 Y0, Y1, YL, YB, YBOT, YBOTTOM, YC, YCENT, YCNTR, YCENTER

SET 3 Z0, Z1, ZL, ZB, ZBOT, ZBOTTOM, ZC, ZCENT, ZCNTR, ZCENTER

SET 4 X2, XR, XRIGHT

SET 5 Y2, YR, YF, YT, YTOP

SET 6 Z2, ZR, ZT, ZTOP.

An additional degree of flexibility is provided by allowing each of the above figures to be displaced or rotated about each of the coordinate axis. The displacement distance is specified by

XCC=x1 YCC=y1 ZCC=z1 (3D)

where the occurence of one, two, or all of the above would displace the geometric frame of reference by x1 in the X coordinate direction, y1 in the Y coordinate direction, or z1 in the Z coordinate direction.

Rotation of the geometric figure can be accomplished by

ANGLA=a DANA=da NDA=1

ANGLB=b DANB=db NDB=m (3-D only)

ANGLC=c DANC=dc NDC=n (3-d only).

Rotation will be accomplished with respect to the geometric frame of reference defined by XCC, YCC, and ZCC. A positive rotation is counterclockwise when viewed along the axis of rotation. All of these, any pair, or any singly defined angle could be defined for rotating about the Z, Y, or X axis by angles a, b, and c degrees, respectively. The DANA=da NDA=1 portion, when present, indicates a replication of the Figure 1 times at increments of da degrees. All of these variables, for specification of displacement or rotation, default to zero. An illustration using a HULL input tape is shown below.

```
GENERATE FIREIN HULL TAPE=AA1
PACKAGE FE RECTANGLE XL=0  XR=10
                     YB=0  YT=1
                     YCC=-.5  XCC=-5  ANGLA=45
```

would result in producing a mask (for a 2-D mesh) in a slab, of thickness 1 cm, centered on the origin, at an angle of 45 degrees to the X coordinate axis. Data would be obtained from the HULL problem on TAPE AA1 and any iron (FE) properties available, as seen through the mask, would be inserted in the new mesh. Subsequent PACKAGE input geometries would obtain data from the same tape until terminated by ENDFIRE.

The most common usage of the package contruct is:

```
PACKAGE material [U=u, V=v, W=w, I=i, RHO=ρ]
    geometry DELETE geom 1 geom 2 —— geom 9.
```

The definition of the material and geometry parameters is as before. The parameters enclosed by [---] and the word DELETE are optional. Any of them may appear. They are intended to specify:

u=X component of material velocity (cm/sec)
v=Y component of material velocity (cm/sec)
w=Z component of material velocity (cm/sec)
i=internal specific energy (ergs/gm)
ρ=material density (gm/cm^3).

The velocity components default to zero. The energy and density default to ambient conditions for standard temperature and pressure, as defined by the equation of state for that material.

Figure 3 is a two-dimensional example with a plot illustrating the effects of the rotation.

A final use of the geometry specifications is the generation of Lagrangian trace particles. This construct is:

PARTICLES Geometry DELETE geom 1, geom 2, --- geom 9.

The description geometry and geom 1 - geom 9 are treated as before, but are used to insert one Lagrangian particle in each cell whose center lies within the region defined. The word DELETE is optional. The particles are counted as they are inserted to keep the ZBLOCK variable NOP properly set.

The final KEEL input set is station generation. This is of the form

```
STATION XS=x1  x2  x3 --- xl
        YS=y1  y2  y3 --- ym
        ZS=z1  z2  z3 --- zn
```

any number of such sets can appear. They have the effect of producing data collection points at each location specified by xi, yj, zk in the list above. This is an implied loop that is executed as

```
LOOP z=z1, z2, z3 --- zn
LOOP y=y1, y2, y3 --- ym
LOOP x=x1, x2, x3 --- xl
ENDLOOP DEFINE station with coordinates (x,y,z).
```

The maximum of l, m, n must be less than NSTN as input to KEEL. The use of XS, YS, ZS (3-D) indicates stations that are to remain fixed with respect to the Eulerian reference. Lagrangian stations, stations that follow the flow, can be inserted by replacing the S in XS, YS, and ZS by an L. A station can have Lagrangian or Eulerian behavior in each of the coordinate directions, i.e., XL, YS, ZL is permissible.

```
GENERATE

PACKAGE FE    CIRCLE XC=24  YC=1.2  RADIUS=1.2
              RECTANGLE  YB=1.2

PACKAGE FE    RECTANGLE  XL=22.8  XR=25.2
                         YB=1.2   YT=40

PACKAGE RHA   V=1.5E5
              RECTANGLE  YB=-8.4  YT=0
              XCC=24  YCC=-2.6
              ANGLA=60
```

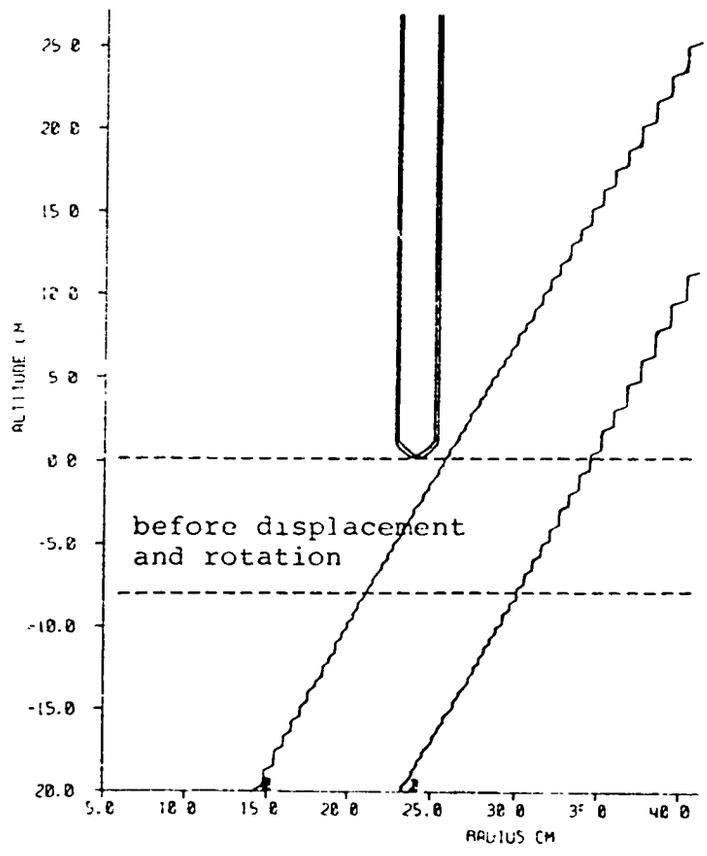

Figure 3. Plot Illustration Rotation

After all particles, stations, and packages have been processed, KEEL writes out the mesh data on unit 4 and the station data on unit 9. The formats are shown in the accompanying Figures 4 and 5. KEEL also performs checks for empty zones and indicates their location and number. A column list and material and particle/station print plot is also produced to allow the user to check the results of generation.

UNIT 4 FORMAT

(DUMP TAPE)

	⌈ 555., PROB, CYCLE, T	(Header Synchronization)
	ZBLK(100,2)	(ZBLOCK Description)
	X(IMAX), YO, Y(JMAX), ZO, Z(KMAX)	(Coordinate Description)
	H (NVARPB)⌉	(Cell by Cell State Variables)
DUMP	.	
	. NBLKS	
	. ⌋	
	H(NOP*NPP) IF NOP>0 and NOP*NPP ≤ NVARPB	(Particle and Station Coordinates)
	⌊H(INT(NVARPB/NPP)*NPP) NOP*NPP ≥ NVARPB	
	.	
	. Additional dumps	
	.	
	666., 666., 666., 666.	(End of the World Trailer ID)
	EOF	
	EOF	

Figure 4. HULL Mesh Dump Tape Format

UNIT 9 FORMAT

(STATION TAPE)

	⌈STAT, PROB		
Header Information	ZBLK (100,2)		
	H(NOP*NPP) NOP*NPP≤1020 otherwise ⌊H(NPP*INT(1020/NPP)) for number of records needed		
	⌈Time (last 15 bits holds number of words at this time)		
	HYDRO P[STA-last 15 bits] Density[Material Code]	STRESS(2-D) Density[STA-last 15 bits] X[Material Code]	STRESS (3-D) Density[STA-last 15 bits] X[Material Code]
	U	U	U
	V	V	V
			W
Repeated for next active station till 1024 word buffer is filled	W	Û	Û
		V̂	V̂
			Ŵ
		SRR-P	SXX-P
		SRZ	SYY-P
		SZZ-P	SZZ-P
		SHOOP-P	SXY
		ERRL	SXZ
		EZZ	SYZ
		EHOOP	EXX
		Y	EYY
		I(NVARST>15)	EZZ
			EXY
			EXZ
			EYZ
			Y
	⌊		Z

MATERIAL CODE - bit number indicates material present i.e., last 15 bits for indicating materials bit on indicates presence of that material = material 3 and 5 is 00000 00000 10100.

Figure 5. HULL Station Tape Format

SECTION V
PROGRAM HULL

HULL solves the finite difference equations on a mesh defined by KEEL or a previous HULL run. The mandatory input is the initial conditions contained on unit 4. If station data is being retained (NSTN>0), then a unit 9 data will also be input. All other input data for running HULL will be obtained from the input data stream. The HULL input must start with the keyword HULL. All data on the HULL input record is in free format, delimited by blank, comma, and equal (=). The HULL input record is logically divided into three parts: restart information, ZBLOCK and program control, and solid material specifications. Figures 6 and 7 are schematic representations of the data flow in program HULL.

A. RESTART DATA

The first portion of the input record immediately following the keyword HULL is used to assure the correct unit 4 input is being read. The parameter name/value pair PROB = XXXX.YYYY is compared with the header record on unit 4. If they are not the same to within 10^{-4}, HULL terminates abnormally. If this test is passed, HULL checks to see if the problem time and cycle number on unit 4 is less than the value desired for restart. If they are not, the unit 4 data are read until the condition is met or the end of file, as designated by the HULL trailer, is reached. If end of file is reached, HULL backspaces unit 4 to the beginning of that dump or time snap-shot for restart. The time and cycle for restart are defaulted to $1.x10^{20}$. If restart at a particular time or cycle is desired, the parameter name/value pairs:

T=t CYCLE=c

can be inserted in the input stream immediately after the PROB=XXXX.YYYY pair. Only T or CYCLE need be input. The value t instructs HULL to restart on the first problem dump whose problem time is greater than or equal to t. Cycle starts are determined in the same way except the comparison is done between the value c and CYCLE as it appears on unit 4. To be used for restart, all of

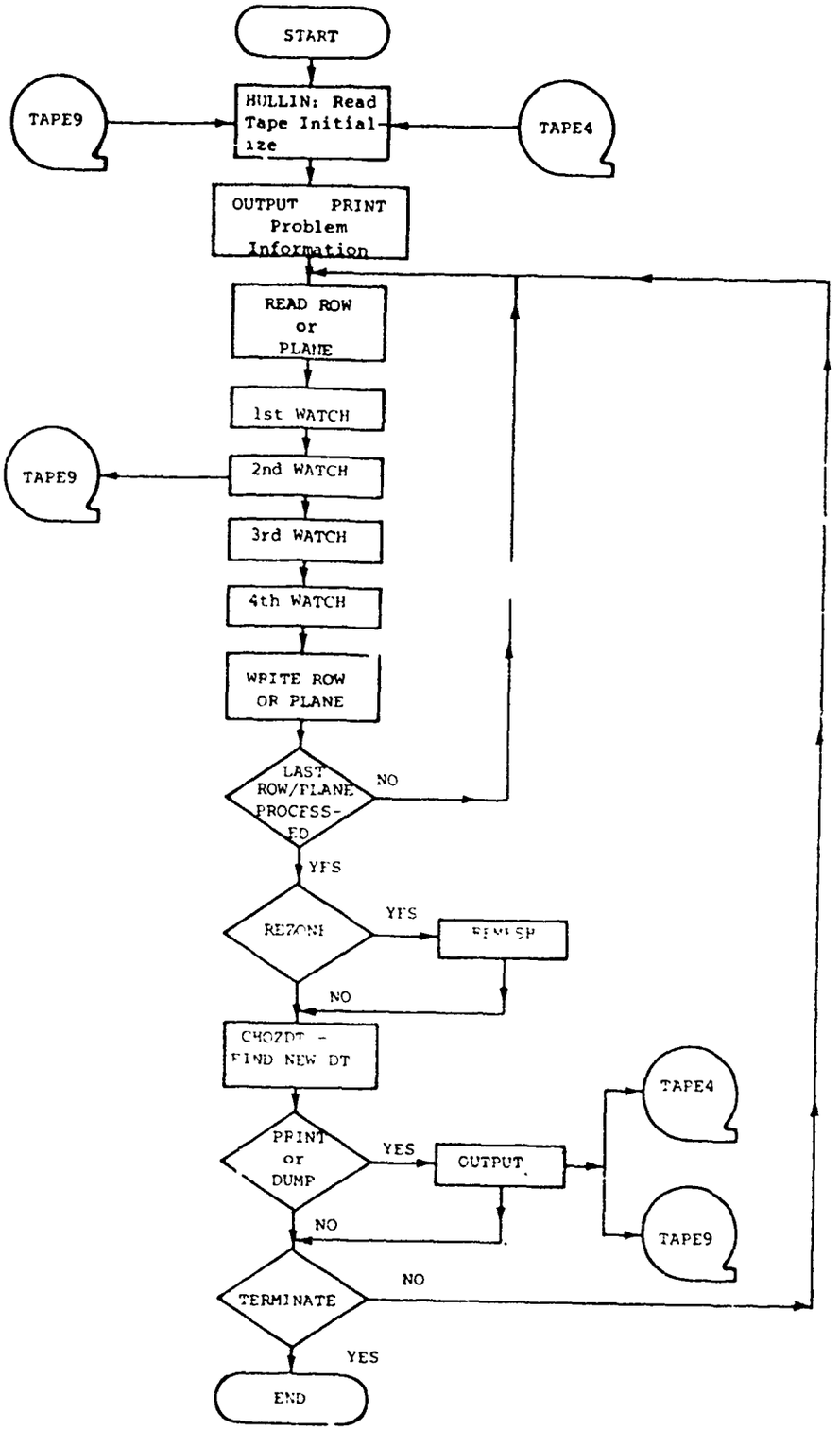

Figure 6. HULL Program Data Flow

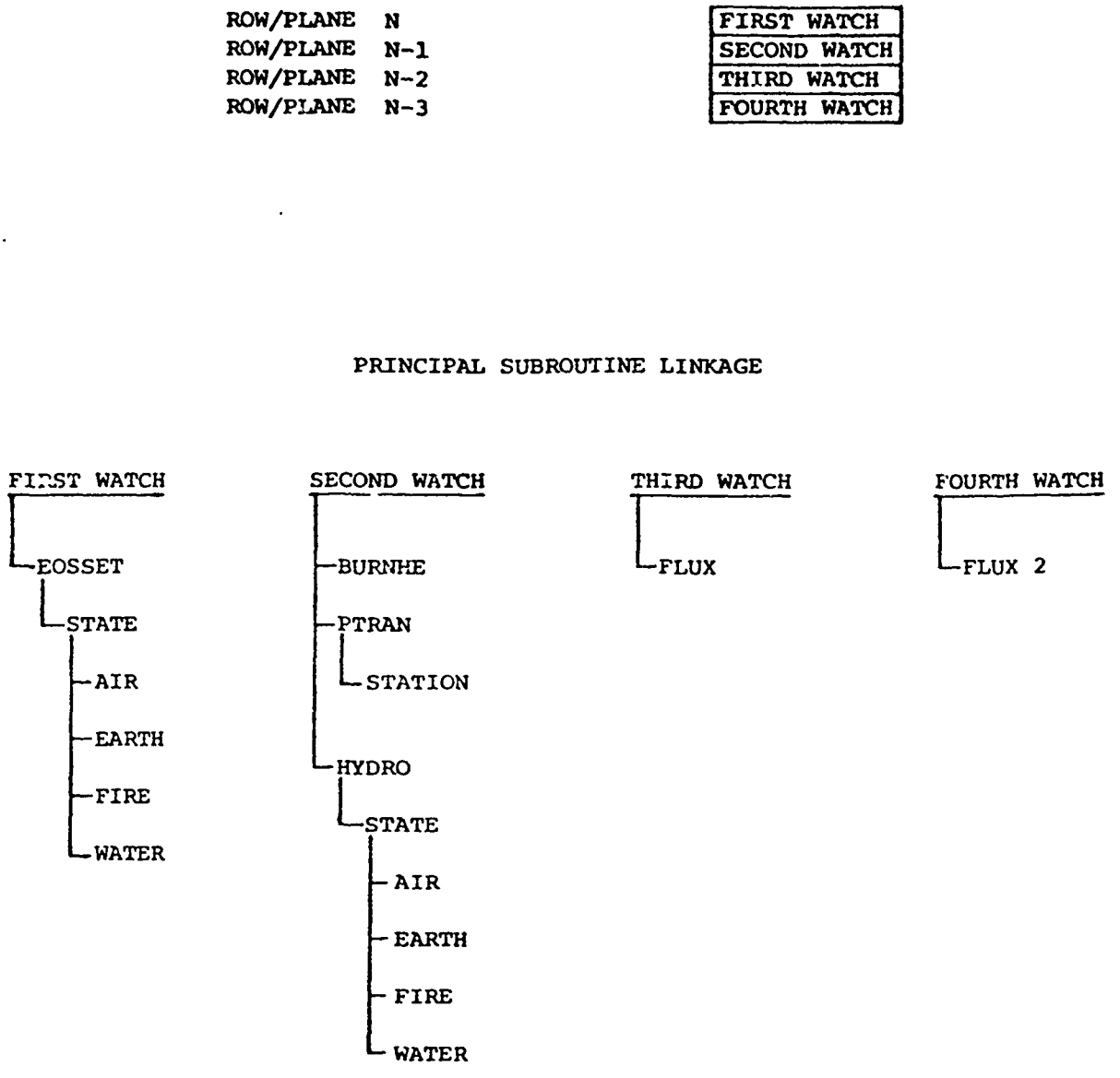

Figure 7. Memory Resident Logical Data Mapping and Principal Subroutine Linkage During In-Cycle Processing

the names PROB, T, and CYCLE must appear before the end of this part of the HULL input record. This part is terminated by the keyword INPUT.

A typical restart segment of a HULL input deck is:

```
HULL PROB=1.001     CYCLE=105
INPUT
```

This would cause HULL to read the unit 4 header, assuring that the tape contained information for problem number 1.001 and advance unit 4 to the first dump whose cycle is greater than or equal to 105. If cycle 100 is the last cycle on unit 4, it would be positioned to restart at cycle 100.

B. ZBLOCK AND PROGRAM CONTROL

The part of the HULL input data following INPUT and preceding the end of file or the keyword MATPROP is used to redefine some ZBLOCK parameters and to specify program edit control and tape dumps. Any parameter value in the ZBLOCK can be modified by this portion of the input stream by using the appropriate ZBLOCK name/value pair in free format as:

```
NAME=value
```

where NAME is the ZBLOCK name and value is an integer, floating point, or logical input definition, as defined in program KEEL. Care should be exercised in this usage since it is possible to redefine the problem number, current time, or cycle by inadvertently placing the keyword PROB, T, or CYCLE after the keyword input rather than before. Thus:

```
HULL PROB=1.001
CYCLE=24
INPUT
T=1.E-6
```

would result in starting the calculation at the first cycle ≥ 24 and then redefining the problem time for that cycle as 1.E-6 seconds. The following ZBLOCK parameters should not be modified since they will change the process flow of the code or the meaning of an edited output variable.

Dangerous HULL INPUT ZBLOCK Redefinitions

SET BY KEEL TO DEFINE THE RUN	SET BY HULL TO PROVIDE INFORMATION
CODE	CYCLE
DIMEN	ELC
IMAX	ETH
JMAX	MLC
KMAX	MTH
NH	SUME
NHIST	T
NM	TLC
NOP	TTIME
NROWPB	TTIME6
NPLPB	TTIME7
NPP	
NSTN	
NVARST	
PROB	

Resetting the values of IQ, JQ, KQ, BURN, FLUXER, FAIL, EOS, or STABF will produce results which may be harmful to the health of the calculation. They should also be used with care.

There are several variables which HULL uses for control during a calculation that do not appear in the ZBLOCK. These are listed along with their function and default values in the tabulation that follows.

Non-ZBLOCK HULL Input Control Parameters

NAME	FUNCTION	DEFAULT VALUE
DCYCST=n	stop calculation in n cycles	$1.x10^{20}$
RTSTOP=t	stop calculation after this run accumulates t CPU hours	$1.x10^{2}$
DEBUG=L	print and dump every cycle if L is .TRUE.	.FALSE.
TIMES=n	n=1 produces 36 dumps per decade in time	1
	n=2 produces dumps at 10ms, 30ms; every .1s from .1s to 1.2s; every 0.5s from 1.5s to 6.0s	
	n=3 produces dumps at interval DMPINT (commonly used for most conventional weapons problems)	

DMPINT=Δt	time interval Δt for producing dumps	$1.x10^{20}$
VMIN=v	minimum velocity component permitted; if the absolute value of any calculated velocity component is less than v it is zeroed	1 cm/sec
ITRACE=i JTRACE=j KTRACE=k	cell (i,j,k) will be traced by printing all intermediate values when SAIL option DEBUG=2 is set	$1.x10^{4}$
HEADER	80 columns from the following card will be used to replace the ZBLOCK header titie	
MRELER	maximum relative error allowed per cycle in mass or energy	$1.x10^{-11}$

This section of HULL input terminates with the keyword MATPROP or an end-of-record.

C. MATERIAL SPECIFICATIONS

All material properties for HULL are defaulted to the best current guess. If a calculation involves a particular material specimen for which experimental results are available, it is, in general, best to replace the default values of the yield strength parameters by their experimentally determined values. The parameters that can be changed through the material properties specification input that follow MATPROP are in a material dependent format that can contain the following optional name/value pairs.

```
MATPROP

MAT=n, YLDST=y1, YLDMAX=y2, EPLAST=e, IFRAC1=if1, YFRAC1=yf1,
IFRAC2=if2, YFRAC2=yf2

ENDPROP
```

If the keyword MATPROP appears, then ENDPROP must appear. The specification of each set of specifications for a given material must be preceded by MAT=n where n is the material number to be changed. The other variable name/value pairs are optional, but names must be spelled correctly or the routine will search forever trying to recognize them. In order for EPLAST=e to have an effect, the ZBLOCK must contain WORK $\geq$1; otherwise, the yield surface will be

defined by YLDMAX. When WORK is on ($\geq$1), plastic work (ε_p) is accumulated for each zone which is occupied by a solid. The yield surface is then defined by Y as a function of the accumulated plastic strain, as indicated in the equation of state description. Yield is always a function of internal specific energy. Thermal softening is accomplished by a trilinear fit as previously discussed in the equation of state section. The thermal factor is multiplied times the current yield surface definition (including plastic hardening) to obtain a final value of Y (flow stress) against which the yield test is made. If you do not input these values, the default values from Table B-2, Appendix B, will be used.

SECTION VI
PROGRAM PULL

Program PULL accepts the calculational mesh data from unit 4 or the station data from unit 9 and produces plotted output on a variety of graphics display devices. The specific plots desired can be specified by the user through the PULL data input stream. INPUT to PULL is free format with input parameter name/value pairs and keywords delimited by blank, comma, or equal (=). The PULL input must start with the word PULL. Actually two different program sets can be produced by SAIL with the keyword PULL. The first is the general mesh data dump plotter which produces plots from the HULL unit 4. The second program set is for sorting and plotting the station tape data written by HULL on unit 9. Program PULL data flow is depicted by Figure 8.

A. DUMP TAPE PLOTS

PULL is capable of making three basic types of plots. A field variable (density, pressure, energy, etc.) versus one of the spatial coordinates (x,y, or z) is called a histogram and is abbreviated for use in the keyword structure as HST. Lines of constant value of a field variable in a two-dimensional representation are plotted for up to 20 arbitrary constants or contours. These are called contour plots and are abbreviated in the keyword calls by CONT. Plots of vector quantities with an arrow indicating vector direction and the arrow-length indicating vector magnitude can be plotted for all vector quantities such as velocity and gradients of scalars. These plots are available only for two-dimensional planes. Vector plots are abbreviated in keywords as VECT. When gradients of scalars are selected, both a vector representation of the gradient and contour plots of the gradient magnitude are produced. Gradient plots are abbreviated in keywords as GRAD.

Histograms can be obtained in any coordinate direction. Those in the x coordinate direction are called horizontal histograms and are abbreviated by H which precedes the designation HST. Histograms in the y or z coordinate direction are called vertical histograms. They are abbreviated by V for construction of keywords. Thus, to obtain histogram plots of any field vari-

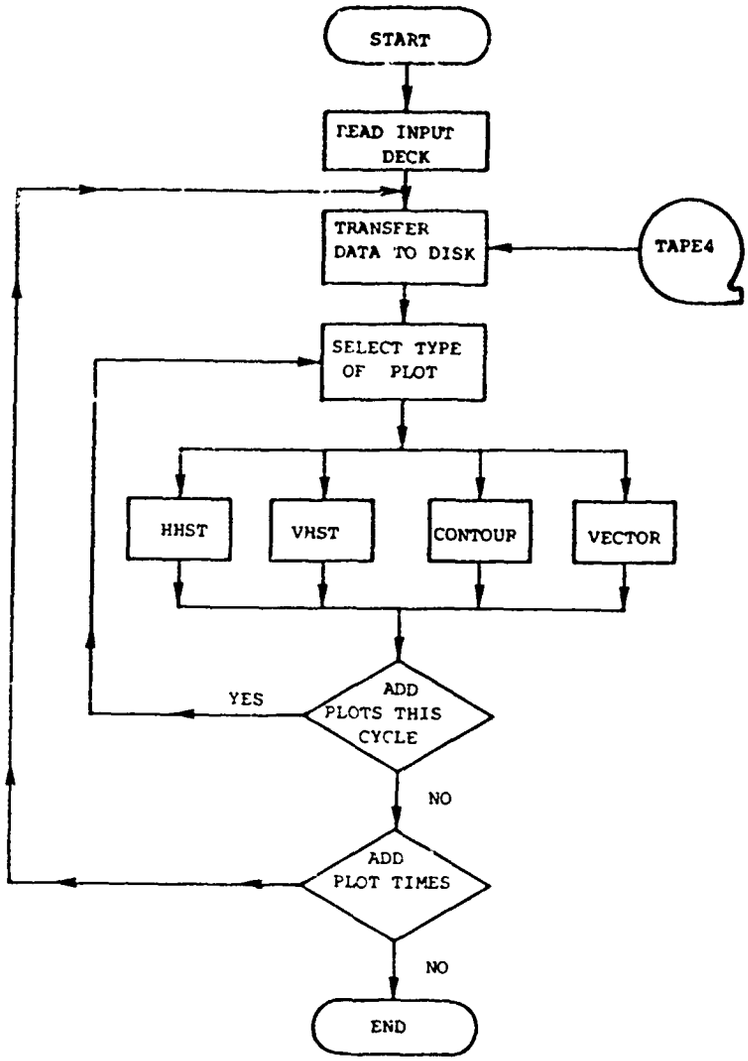

Figure 8. Program PULL Data Flow

able XXX the keyword would be XXXVHST or XXXHHST. Contour plots are called by XXXCONT and gradient plots are selected by XXXGRAD. The allowable designators, XXX and their meanings are tabulated in Table 1.

TABLE 1. PLOT DESIGNATORS

XXX	DEFINITION	POSSIBLE PLOTS HHST	VHST	CONT	GRAD
P	HYDROSTATIC PRESSURE	X	X	X	X
U	X COMPONENT OF VELOCITY	X	X	X	X
V	Y COMPONENT OF VELOCITY	X	X	X	X
W(3D)	Z COMPONENT OF VELOCITY	X	X	X	X
E	SPECIFIC INTERNAL ENERGY	X	X	X	X
D	DENSITY	X	X	X	X
T	TEMPERATURE	X	X	X	X
M	INDIVIDUAL MATERIAL DENSITY	X	X	X	X
K	SPECIFIC KINETIC ENERGY	X	X	X	X
RP	PRESSURE IN ATMOSPHERES	X	X	X	X
RE	ENERGY IN UNITS OF AMBIENT	X	X	X	X
RD	DENSITY RELATIVE TO AMBIENT	X	X	X	X
SXX(3D)	X COORDINATE NORMAL STRESS DEVIATOR	X	X		
SYY(3D)	Y COORDINATE NORMAL STRESS DEVIATOR				
SRR(2D)	RADIAL STRESS DEVIATOR	X	X		
SZZ	Z OR AXIAL STRESS DEVIATOR	X	X		
SHH(2D)	HOOP DEVIATORIC STRESS	X	X		
SRZ(2D)	SHEAR STRESS	X	X		
SXY(3D)	SHEAR STRESS	X	X		
SXZ(3D)	SHEAR STRESS	X	X		
SYZ(3D)	SHEAR STRESS	X	X		
S2I	SECOND STRESS INVARIANT	X	X	X	
SP1	MAXIMUM PRINCIPAL STRESS	X	X	X	
SP2	MINIMUM PRINCIPAL STRESS	X	X	X	
EXX(3D)	STRAIN FOR SXX	X	X		
EYY(3D)	STRAIN FOR SYY	X	X		
ERR(2D)	STRAIN FOR SRR	X	X		
EZZ	STRAIN FOR SZZ	X	X		
EHH(2D)	STRAIN FOR SHH	X	X		
ERZ(2D)	STRAIN FOR SRZ	X	X		
EXY(3D)	STRAIN FOR SXY	X	X		
EXZ(3D)	STRAIN FOR SXZ	X	X		
SYZ(3D)	STRAIN FOR SYZ	X	X		
E2I	SECOND STRAIN INVARIANT	X	X	X	
EP1	MAXIMUM PRINCIPAL STRAIN	X	X	X	
EP2	MINIMUM PRINCIPAL STRAIN				

Thus, PCONT would be the allowable keyword for selecting pressure contours.

As many of the above as have meaning (i.e., SRR is not available for STRESS=0 run) can be entered for a single FULL run.

Contour plots can be modified by a list of variables following the keyword to specify contour values to be plotted. For example:

DCONT = 1 1.5 2 3 4 5

instructs PULL to draw contour lines for each of the density values 1, 1.5, 2, 3, 4, and 5 gm/cm^3. Contours can also be modified by ICTRS=n and CONYV=x1, x2, ..., xn being selected for the contour plot requests that follow. The value of n can be from 2 to 20; default is 10. The values of the x_i can be any real number. The default is a set of contours equally spaced from the minimum to the maximum in n steps. In addition, contour values can be modified by the keywords LOGV and LOGD. LOGV causes all contour values to be set to values of $1x10^n$ and $3x10^n$ from the maximum value down for all decreasing integers below n until the number of contours is exhausted. LOGD produces the contours of logarithm base 10 of the data. Contours are plotted by interpolating the data and smoothing to second order. This can be turned off by including the keyword NOCURV. The inclusion of the keyword COORD will cause the zone size and the values and coordinates of the minimum and maximum to be printed on each plot. If particles or stations were included in the calculation (NOP$\geq$1), the particle positions can be presented as dots on the contour plots. This is obtained by including the keyword PART in the input deck.

Histograms will be produced at the mesh boundary lower limit as a default. By specifying:

PVHST = x1, x2, dx

histograms will be produced along all vertical columns starting at x1 with increment dx to x=x2.

In order to get the above equivalent plots from a 3-D calculation, it is necessary to specify XPLANES, YPLANES, or ZPLANES followed by 3 values as histograms were described above. The following contour specifications will

then be satisfied in those planes. If the keyword 3DPLOTS is present for a 3D problem, then the user may specify RDCONT, RECONT, or RPCONT to plot a 3D representation of the selected contour values when viewed in the plane normal to a line from the mesh origin to the position EYE=x, y, z. The contours will be considered opaque for this representation.

The problem times at which plots can be selected are set by:

CTIME=t1, t2, ... tn
DTIME=dt
FTIME=t1
LTIME=tn

If CTIME is present, it must be the last data in the KEEL input stream. Plots will be produced for all times t1, t2, ..., tn. The remaining three time selectors will produce plots from all dumps after t1 at time intervals of dt to the last time $\leq$ tn. If dt is not specified, all dumps between t1 and tn will be plotted. If the keyword STIME is included, only standard times (TIMES=1 from HULL specification) will be produced.

B. STATION PLOTS

Plots of the station parameters can be obtained by following the keyword PULL by the keyword STATION. Sorting and plotting routines are written to file SAIL. They must be compiled separately and in sequence. The first compilation must be executed first to produce an unpacked station file with all time data for station one first, followed by all data for station two until station NSTN is reached. If the HULL calculation that produced the file was a pure hydrodynamic run (STRESS=0), then the execution of the second compilation will plot only the pressure, density, dynamic pressure, and impulse as a function of time. The stations to be plotted can be determined by FSTA=n1 LSTA=n2. All stations from n1 to n2 will be plotted. The defaults are n1=1 and n2=NSTN. If STRESS$\geq$1, then a specific set of plots can be requested. These are:

RADIAL	- produce plots of radial motion
AXIAL	- produce plots of axial motion
TOTAL	- produce plots of total motion
STRESS	- produce plots of each stress component
STRESSDEV	- produce plots of each stress deviator component
TENSOR	- produce plots of the 2nd stress invariant and the principal stresses
FLUXM	- produce a plot of mass flux

We have thus far not mentioned the use of the problem number for PULL input. It is not needed unless it will be used to find the input files from the tape library or the permanent file directory. If PROB=XXXXX.YYYY is present, it must be correct for this run or PULL will terminate abnormally.

SECTION VII

RUNNING A CALCULATION

The preceding sections have addressed set-up of the various input data streams for running programs in the HULL system. We have not discussed system considerations because of the frequent changes made in operating systems. HULL is most often used on CDC operating systems. We therefore follow with an example control card and data deck for setting up, running, and plotting a HULL calculation. We will assume the existence of HULLIB, a user library containing the absolutes of programs SAIL and PLANK, and relocatables for the HULL prologue routines on the HULL system file.

A. KEEL RUN DECK

The deck below will set up 50 (X coordinate) by 100 (y coordinate) zones with DX=DY=.1 cm from X_0=0 and Y_0=-5, respectively. The materials will be air, iron (FE), and copper (CU). Copper will be placed in a cylinder of radius 1 cm extending from Y=0 to Y=5. Iron will be placed in a slab from Y=0.5 to Y=0. The iron slab is given an initial velocity of 2.10^5 cm/sec.

```
ATTACH (HULLIB,ID=local)
LIBRARY (HULLIB)
LABEL (TAPE4,W,T=0,VSN=MINE)
PLANK.
ATTACH (OLD,HULLSYSTEMFILE,ID=local)
SAIL.
FTN (I=SAIL,B=KEEL,OPT=2,L=0)
KEEL.
*EOR or 7/8/9 for card input deck
KEEL PROB=1.
IMAX=50  JMAX=100  DIMEN=2  STRESS=1
NM=3  AIR=1  FE=2  CU=3
REZONE=0 Header
         HULL EXAMPLE PROBLEM
MESH X0=0  XMAX=5  YO=-5  YMAX=5
GENERATE
PACKAGE AIR RECTANGLE
        DELETE RECTANGLE XR=1  YB=0
               RECTANGLE YB=0.5 YB=0
PACKAGE CU RECTANGLE XR=1  YB=0
```

```
PACKAGE FE V=2.E5
        RECTANGLE YB=-5 YT=0
END
*EOR
SAIL LINENO
```

Change cards needed by this run

B. HULL RUN DECK

```
ATTACH (HULLIB,ID=local)
LIBRARY (HULLIB)
LABEL (TAPE4,R,VSN=MINE)
PLANK.
ATTACH (OLD,HULLSYSTEMFILE,ID=local)
SAIL.
FTN (I=SAIL,B=HULL,OPT=2,L=0)
HULL.
*EOR
SAIL LINENO
local changes
*EOR
HULL PROB=1. CYCLE=0
INPUT
TIMES=3  DMPINT=5.E-6
PTSTOP=50.E-6
MATPROP
MAT=2 YLDMAX=15.E9
MAT=3 YLDMAX=4.5E9
ENDPROP
*EOR
```

C. PULL RUN DECK

```
ATTACH (HULLIB,ID=local)
LIBRARY (HULLIB)
LABEL (TAPE4,R,VSN=MINE)
PLANK.
ATTACH (OLD,HULLSYSTEMFILE,ID=local)
SAIL.
FTN (I=SAIL,B=PULL,L=0,OPT=2)
PULL.
*EOR
DCONT=7.8  7.9  8  8.1  8.2  8.3  8.6  8.7  8.8  9
PVHST  VVECT  COORD  NOCURV
STIME
```

SECTION VIII
SENSE SWITCH CONTROL

HULL can be controlled for set up by SAIL and during running by CDC sense switch settings. If the sense switches are set before PLANK runs, they have the following effects:

SENSE SWITCH ON BEFORE PLANK RUNS

SENSE SWITCH NUMBER	EFFECT
0 (All switches off)	ECS store of the mesh
5	In-core central memory storage of the mesh
6	Disk storage of the mesh

If the sense switches are turned on after HULL or PULL have started, the following actions will be taken:

SENSE SWITCH ON AFTER HULL OR PULL START

SENSE SWITCH NUMBER	EFFECT
1	Stop at end of this cycle (HULL)
	plot this dump and stop (PULL)
2	Route current output to printer (both)
3	Display current time, cycle, time step and estimate of running time on operators console (HULL)
	Display time, cycle, and type of plot being made (PULL)
4	Change dump tapes (HULL)
	Change to next input tape (PULL)
5	Start double buffering of a disk storage problem. Increases central memory but develops I/O. (HULL)

5 OFF	Stop double buffering, return unneeded central memory. (HULL)
6	Roll ECS out, resume by operator GO. (HUlL)

The sense switches can be toggled on by the operator at his console or through the use of control cards in the input stream.

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APPENDIX A

BASIC EQUATIONS AND SOLUTION ALGORITHM

The HULL code solves the partial differential equations of continuum mechanics. Explicit terms for heat conduction and viscous effects are not included. The equations solved in finite difference form by HULL are:

$$\dot{\rho} + \rho u_{i,i} = 0 \tag{A-1}$$

$$\rho \dot{u}_j - T_{ij,i} = -\rho g_j \tag{A-2}$$

$$\rho \dot{E} - (T_{ij} u_j)_{,i} = -\rho u_j g_j \tag{A-3}$$

$$T_{ij} = f(\rho, I, u_{i,j} + u_{j,i}) \tag{A-4}$$

where:

ρ = material density (gm/cc)

u_j = material velocity (cm/sec)

$T_{ij} = S_{ij} - \delta_{ij} P$, stress tensor,
P = hydrostatic pressure (dynes/cm²)

S_{ij} = stress deviator (dynes/cm²)

g_j = gravitational body force (cm/sec²)

$E = I + 1/2\ u_j u_j$, total specific energy,
I = internal specific energy (ergs/gm)

subject to: $S_{ij} S_{ij} \leq \frac{2Y^2}{3}$, Y = flow stress.

The deviatoric strain rate tensor can be expressed as:

$$\dot{e}_{ij} = \dot{\varepsilon}_{ij} = (u_{i,j} + u_{j,i})/2 - \delta_{ij} u_{i,i}/3. \tag{A-5}$$

The deviatoric stress tensor is linearly related to the strain tensor by:

$$S_{ij} = 2Ge_{ij} \tag{A-6}$$

where G is the elastic shear modulus.

Plasticity is modeled by the Von Mises yield criterion:

$$S_{ij}\, S_{ij} \leq \frac{2Y^2}{3},\ Y = \text{tensile flow stress.}$$

If the stress deviators are not contained within this measure, they are brought back to the yield surface by:

$$S_{ij} = S_{ij} \left[\frac{2Y^2}{3S_{ij}S_{ij}} \right]^{1/2} \tag{A-7}$$

The initiation of failure can be determined by one of three criteria:

(1) Maximum principal stress can be used as a crude means to determine if a spall threshold or failure in plane strain has occurred. The principal stresses are defined by solving:

$$|T_{ij} - \sigma\delta_{ij}| = 0. \tag{A-8}$$

Failure is initiated whenever:

$$\text{Maximum } [\sigma] \geq F \cdot Y$$

where F is a constant between 2.5 and 4.

(2) Maximum principal strain can also be used to determine the onset of failure in plane stress or ductile failure by solving the analog of Equation (A-8) for the maximum principal strain as:

$$|\varepsilon_{ij} - \lambda\delta_{ij}| = 0 \tag{A-9}$$

and assuming failure occurs whenever

$$\text{Maximum } [\lambda] \geq e_f$$

where e_f is the tensile strain to failure of the specific material.

(3) Triaxial states at which failure can occur are based on a P/Y model (Reference 6). In this formulation failure is initiated when the maximum principal strain (λ, Equation (A-9)) exceeds a failure surface $\lambda_f = f(P/Y)$ which is determined by experiment. Failure in HULL is initiated by inserting a minimum numerically significant void in the failed zone and allowing the tensile forces to relax. Subsequent tension or shear of this region is not supported (i.e., the zone defines a slip surface and tensile stresses are zeroed).

Rewriting Equations (A-1) through (A-3) in axisymmetric cylindrical coordinates;

$$\dot{\rho} + \rho\left[\frac{1}{r} \frac{\partial(ru)}{\partial r} + \frac{\partial v}{\partial z} \right] = 0 \qquad \text{(A-10)}$$

$$\rho\dot{u} - \frac{\partial_r T_{rr}}{r\partial r} - \frac{\partial T_{rz}}{\partial z} + \frac{T_{\theta\theta}}{r} = 0 \qquad \text{(A-11)}$$

$$\rho\dot{v} - \frac{\partial_r T_{rz}}{r\partial r} - \frac{\partial T_{zz}}{\partial z} = -\rho g \qquad \text{(A-12)}$$

$$\rho\dot{E} - \frac{\partial}{r\partial r}\left[r(uT_{rr} + vT_{rz}) \right]$$

$$- \frac{\partial}{\partial z}(uT_{rz} + vT_{zz}) = -\rho v g \qquad \text{(A-13)}$$

r, z = radial and axial coordinates

u = radial velocity component

v = axial velocity component

$T_{rr} = S_{rr} - P$

$T_{zz} = S_{zz} - P$

$T_{\theta\theta} = S_{\theta\theta} - P$

$T_{rz} = T_{zr} = -S_{rz}$

$$\dot{S}^e_{rr} = 2G(\dot{\varepsilon}_{rr} - D)$$

$$\dot{S}^e_{zz} = 2G(\dot{\varepsilon}_{zz} - D)$$

$$\dot{S}^e_{\theta\theta} = -\dot{S}_{rr} - \dot{S}_{zz}$$

$$\dot{S}^e_{rz} = 2G(\dot{\varepsilon}_{rz})$$

$$\dot{\varepsilon}_{rr} = \frac{\partial u}{\partial r}\ ,\ \dot{\varepsilon}_{zz} = \frac{\partial v}{\partial z}\ ,\ \dot{\varepsilon}_{rz} = \left[\frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}\right]/2$$

$$D = \left[\frac{\partial ru}{r\partial r} + \frac{\partial v}{\partial z}\right]/3$$

Subject to the yield criteria:

$$S_{\alpha\beta} = S^e_{\alpha\beta};\ J = (S^2_{rr} + S^2_{rz} + S^2_{zz} + S_{rr}S_{zz}) \leq \frac{Y^2}{3}$$

$$= S^e_{\alpha\beta}\left[\frac{Y^2}{3J}\right]^{1/2} \qquad J > \frac{Y^2}{3}$$

Failure implies that $\sigma \geq \sigma_{frac}$, where:

$$\sigma = 1/2(T_{rr} + T_{zz}) + \left[\left(\frac{T_{rr}-T_{zz}}{2}\right)^2 + T_{rz}2\right]^{1/2}$$

and σ_{frac} is the tensile yield stress, a number usually between 2 and 5 times the flow stress.

DIFFERENCE METHOD (2D)

Equations (A-10), (A-11), (A-12), and (A-13) are solved on a finite difference mesh of discrete intervals Δr_i ,Δz_j in the radial and axial coordinates, respectively. The solution is advanced explicitly from the initial conditions by discrete time steps Δt^n. The solution is thus defined

on the mesh (r_i, z_j, t^n) where each of the variables f(r,z,t) in the solution space defined by

$$f^n_{i,j} = f(r_i, z_j, t^n)$$

where

$$r_i = r_o + \left[\sum_{k=1}^{i-1} \Delta r_k\right] + \frac{\Delta r_i}{2} \tag{A-14}$$

$$z_j = z_o + \left[\sum_{k=1}^{j-1} \Delta z_k\right] + \frac{\Delta z_j}{2} \tag{A-15}$$

$$t^n = t_o + \sum_{k=1}^{n} \Delta t^n \tag{A-16}$$

It should be noted that this subscript notation is distinct from the previous tensor notation.

r_o, z_o, t_o are the initial points in the r, z, and t coordinates, respectively. We also find it convenient to define $\Delta r_{i+1/2} = r_{i+1} - r_i$ and $\Delta z_{j+1/2} = z_{j+1} - z_j$.

The variables are defined for each (r_i, z_j) at time t^n. Spatial derivatives will be approximated by the finite difference operators δr and δz as:

$$\delta^r (f^n_{i,j}) = (f^n_{i+1/2,j} - f^n_{i-1/2,j})/\Delta r_i$$

$$\delta^z (f^n_{i,j}) = (f^n_{i,j+1/2} - f^n_{i,j-1/2})/\Delta z_j$$

Time derivatives will be approximated by $(f^{n+1}_{i,j} - f^n_{i,j})/\Delta t$ where $\Delta t = \Delta t^n$ is assumed. It should be noted that the time derivatives appearing in Equations

(A-10) through (A-16) are Lagrangian; that is, they refer to a coordinate system moving with the material. Hence the $f^{n+1}_{i,j}$ values refer to the property f at some displaced location. This must be borne in mind, as it is not noted explicitly in the equations which follow:

The solution for (A-11), (A-12), and (A-13) is thus:

$$u^{n+1} = u^n - \frac{\Delta t}{\rho^n}\left[\ \delta^r p^{n+1/2} - \frac{1}{r}\delta^r r S^n_{rr} - \delta^z S^n_{rz} + \frac{S^n_{rr}+S^n_{zz}}{r}\ \right] \qquad \text{(A-17)}$$

$$v^{n+1}_{i,j} = v^n + \frac{\Delta t}{\rho^n}\left[\ \delta^z(S^n_{zz} - p^{n+1/2})\ \frac{1}{r}\delta^r(rS^n_{rz})\ \right] \qquad \text{(A-18)}$$

$$E^{n+1} = E^n + \frac{\Delta t}{\rho^n}\left[\ \frac{1}{r}\delta^r[ru^{n+1/2}(S^n_{rr} - p^{n+1/2}) + rv^{n+1/2}S^n_{rz}] \right. \qquad \text{(A-19)}$$

$$\left. \delta^z[v^{n+1/2}(S^n_{zz} - p^n) + u^{n+1/2}S^n_{rz}]\ \right]$$

Since the pressure is a function of density and internal energy only, we can obtain a substantive time derivative for pressure by

$$\dot{P} = \frac{\partial P}{\partial \rho}\dot{\rho} + \frac{\partial P}{\partial I}\dot{I} \qquad \text{(A-20)}$$

From Equations (A-11) through (A-13) we can obtain:

$$\dot{\rho I} + Pu_{i,i} = 0 \qquad \text{(A-21)}$$

Combining Equations (A-1), (A-20), and (A-21) we can write

$$\dot{P} = -\rho\left[\ \frac{\partial P}{\partial \rho} + \frac{P}{\rho^2}\frac{\partial P}{\partial I}\ \right] v_{i,i}$$

but the term in parentheses is the square of the isentropic sound speed C_s, hence:

$$\dot{P} = -\rho C_s^{\ 2}\ u_{i,i} \qquad \text{(A-22)}$$

Using Equation (A-22) we can calculate

$$p^{n+1/2}_{i+1/2,j} = p^{n}_{i+1/2,j} - \frac{\Delta t}{2} (\rho C_s^2)^{n}_{i+1/2,j} \frac{1}{r_i} \delta^r (ru)^{n}_{i+1/2,j} \qquad \text{(A-23a)}$$

$$p^{n+1/2}_{i,j+1/2} = p^{n}_{i,j+1/2} - \frac{\Delta t}{2} (\rho C_s^2)^{n}_{i,j+1/2} \delta^z (v^{n}_{i,j+1/2}) \qquad \text{(A-23b)}$$

where:

$$p^{n}_{i+1/2,j} = \frac{p^{n}_{i+1,j} \rho^{n}_{i,j} + p^{n}_{i,j} \rho^{n}_{i+1,j}}{\rho^{n}_{i,j} + \rho^{n}_{i+1,j}} \qquad \text{(A-24a)}$$

$$p^{n}_{i,j+1/2} = \frac{p^{n}_{i,j+1} \rho^{n}_{i,j} + p^{n}_{i,j} \rho^{r}_{i,j+1}}{\rho^{n}_{i,j} + \rho^{n}_{i,j+1}} \qquad \text{(A-24b)}$$

$$(\rho C_s^2)^{n}_{i+1/2,j} = \text{Min} \left[(\rho C_s^2)^{n}_{i,j} ; (\rho C_s^2)^{n}_{i+1,j} \right] \qquad \text{(A-25a)}$$

$$(\rho C_s^2)^{n}_{i,j+1/2} = \text{Min} \left[(\rho C_s^2)_{i,j} ; (\rho C_s^2)^{n}_{i,j+1} \right] \qquad \text{(A-25b)}$$

and $(\rho C_s^2)^{n}_{i,j}$ is obtained from the equation-of-state.

The spatial centered pressures of Equation (A-24) were chosen to avoid over-acceleration in regions of large density gradients. This formulation results in correct free surface conditions at solid/air interfaces at one limit and degenerates to a simple average at the other limit of small density variations. Minimum values of ρC_s^2 in Equation (A-25) are used to keep from overpredicting the pressures calculated in Equation (A-23). The physical rationale for this choice is the assumption that the most easily compressed material will restrict the pressure rise since it will compress first. The procedures also switch the scheme from second order accuracy, in regions of smooth flow, to first order accuracy in regions of rapid transitions in a more simplified calculational methodology than described by Harten and Zwas (Reference 3).

Time advanced velocities for Equation (A-19) are obtained by differencing Equations (A-14) and (A-15) by:

$$u^{n+1/2}_{i+1/2,j} = u^{n}_{i+1/2,j} - \frac{\Delta t}{2\rho^{n+1/2}_{i+1/2,j}} \delta r(P^{n}_{i+1/2,j}) \tag{A-26}$$

$$v^{n+1/2}_{i,j+1/2} = v^{n}_{i,j+1/2} - \frac{\Delta t}{2\rho^{n+1/2}_{i,j+1/2}} \delta r(P^{n}_{i,j+1/2}) - \frac{\Delta t}{2} g_{j+1/2}$$

where:

$$g_{j+1/2} = \frac{g_j + g_{j+1}}{2}$$

and:

$$u^{n}_{i+1/2,j} = \frac{\rho^{n}_{i,j}\, u^{n}_{i,j} + \rho^{n}_{i+1,j}\, u^{n}_{i+1,j}}{\rho_{i,j} + \rho_{i+1,j}} \tag{A-27}$$

$$v^{n}_{i,j+1/2} = \frac{\rho^{n}_{i,j}\, v^{n}_{i,j} + \rho^{n}_{i,j+1}\, v^{n}_{i,j+1}}{\rho^{n}_{i,j} + \rho^{n}_{i+1,j}}$$

$$\rho^{n}_{i+1/2,j} = \text{MAX}\ (\rho^{n}_{i,j}\ ,\ \rho^{n}_{j+1,j}) \tag{A-28}$$

$$\rho^{n}_{i,j+1/2} = \text{MAX}\ (\rho^{n}_{i,j}\ ,\ \rho^{n}_{i,j+1})$$

where again the selection of spatially centered quantities in Equations (A-27) and (A-28) was made to reduce overacceleration in regions of large density gradients.

The density Equation (A-10) is also solved in the first step. The new densities are stored and used in the fluxing (or Rezone) calculation below. Equation (A-10) is differenced as:

$$\rho^{n+1}_{i,j} = \rho^{n}_{i,j} \left[\ 1 - \frac{\Delta t}{r_i} \delta^{r}\ (ru)^{n+1/2}_{i,j} - \Delta t\ \delta^{z} v^{n+1/2}_{i,j}\ \right] \tag{A-29}$$

using the definitions above for the time-centered velocities.

The deviators by definition have the property $S_{ii} = 0$ leaving only S_{rr}, S_{rz} and S_{zz} to be retained as stored quantities from the previous time step. These deviators are defined at each mesh point in the solution space. The numerical solution is obtained explicitly by:

$$S\alpha\beta^{n}_{i+1/2,j} = \emptyset_{i+1/2,j} \frac{(S\alpha\beta^{n}_{i,j}\ \rho^{n}_{i+1,j} + S\alpha\beta^{n}_{i+i,j}\ \rho^{n}_{i,j})}{\rho_{i,j} + \rho_{i+i,j}} \qquad (\alpha=r,z;\beta=r,z)$$

$$S\alpha\beta^{n}_{i,j+1/2} = \emptyset_{i,j+1/2} \frac{(S\alpha\beta^{n}_{i,j}\ \rho^{n}_{i,j+1} + S\alpha\beta^{n}_{i,j+1}\ \rho^{n}_{i,j})}{\rho_{i,j} + \rho_{i,j+1}}$$

$$\emptyset_{i+1/2,j} = \text{Min}\ (VF_{i,j}\ ,\ VF_{i+i,j})$$

$$\emptyset_{i,j+1/2} = \text{Min}\ (VF_{i,j}\ ,\ VF_{i,j+1})$$

$VF_{i,j}$ = Fractional volume of solid in zone i,j.

Other quantities required above are defined by Equations (A-17), (A-18), (A-19), and (A-27).

At the end of this step, the stress deviators themselves are updated by:

$$S_{rr}{}^{n+1}_{i,j} = S_{rr}{}^{n}_{i,j} + 2G\ \Delta t \left[\delta^{r}\ (u)^{n}_{i,j} - D\right] \qquad \text{(A-34a)}$$

$$S_{zz}{}^{n+1}_{i,j} = S_{zz}{}^{n}_{i,j} + 2G\ \Delta t \left[\delta^{z}\ (v)^{n}_{i,j} - D\right] \qquad \text{(A-34b)}$$

$$S_{rz}{}^{n+1}_{i,j} = S_{rz}{}^{n}_{i,j} + G\ \Delta t\ (\delta r\ (v)^{n}_{i,j} + \delta z\ (u)^{n}_{i,j}) \qquad \text{(A-34c)}$$

Where G is a material dependent modulus and:

$$D = \left[\frac{\delta r}{r_i} (ru)^{n+1/2}_{i,j} + \delta z \, v^{n+1/2}_{i,j} \right]/3$$

These definitions of the stress deviators are subject to the yield criteria:

$$S_{\alpha\beta} = S^e_{\alpha\beta} \; ; \; J = (S^2_{rr} + S^2_{rz} + S^2_{zz} + S_{rr}S_{zz}) \leq 1/3 \; Y^2$$

$$= S^e_{\alpha\beta} \left(\frac{Y^2}{3J}\right)^{1/2} \qquad J > 1/3 \; Y^2$$

Failure implies that $\sigma \geq \sigma_{frac}$, where:

$$\sigma = 1/2 \; (T_{rr} + T_{zz}) + \left[\left(\frac{T_{rr} - T_{zz}}{2}\right)^2 + T_{rz}^2 \right]^{1/2}$$

and σ_{frac} is the tensile yield stress, a number usually between 2 and 5 used in the HULL code. The modularity of the code makes substitution by more comprehensive failure models an easy matter.

Implementation of failure in a zone is accomplished by inserting a void or gas (air) into the "unoccupied" volume of a cell after allowing the failed solid to relax to ambient density. Use of a gas rather than a void provides a damping mechanism reducing the tendency of a failed cell to "chatter" as it is reloaded.

The definition of a slip surface is the existence of more than one solid (but no gas) in a cell. Such a cell will permit compressive stress but will not support shear or tensile force. Cells which contain a gas or void will not support either distortional or spherical tensile forces. This implies that surfaces across failed regions are automatically defined as slip surfaces whenever a failure criterion is invoked.

DIFFERENCE METHOD (3D)

Equations (A-1) through (A-3) are solved on a finite difference mesh composed of discrete spatial intervals x_i, y_j, z_k in the three-dimensional Cartesian coordinate system having coordinates x, y, and z. The solution is advanced explicitly from the initial conditions by discrete time intervals Δt^n. The solution at any time t^n for any variable f(x,y,z,t) in the solution space is $_n$ defined by $f_{i,j,k} = f(x_i,y_j,z_k,t_n)$ where:

$$x_i = x_o + \sum_{m=1}^{i-1} \Delta x_m + \frac{\Delta x}{2} i$$

$$y_j = y_o + \sum_{m=1}^{j-1} \Delta y_m + \frac{\Delta y}{2} j$$

$$z_k = z_o + \sum_{m=1}^{k-1} \Delta z_m + \frac{\Delta z}{2} k$$

$$t^n = t_o + \sum_{m=1}^{n} \Delta t_m$$

Note that on this mesh $\Delta x_i = x_{i+1} - x_i$.

This definition holds analogously for y, z, and t.

The variables that are defined for each point in the mesh at time t^n and location x_i, y_j, z_k are:

P = pressure (dynes/cm²)
u = x component of velocity (cm/sec)
v = y component of velocity (cm/sec)
w = z component of velocity (cm/sec)
I = internal specific energy (ergs/gm)
M = mass (gms)
M_n = mass of the nth species (gms)
V_n = volume occupied by the nth species (cm³)

I_n = total internal energy of the n^{th} species (ergs)

S_{xx} = x coordinate deviatoric stress (dynes/cm²)

S_{yy} = y coordinate deviatoric stress (dynes/cm²)

S_{xy} = shear stress normal to z coordinate (dynes/cm²)

S_{xz} = shear stress normal to y coordinate (dynes/cm²)

S_{yz} = shear stress normal to x coordinate (dynes/cm²)

Since the deviators have the property

$$S_{zz} = -S_{xx} - S_{yy}$$

S_{zz} is not explicitly stored as a mesh variable.

If we define the central difference operator $\delta^x(f_{i,j,k})$ as:

$$\delta^x(f^n_{i,j,k}) = \frac{(f_{i+1/2,j,k} - f_{i-1/2,j,k})}{\Delta x_i}$$

then we can express the calculation for the velocity components as:

$$u^{n+1} = u^n + \frac{\Delta t}{\rho^n}\left[\ \delta^x(S^n_{xx} - p^{n+1/2}) + \delta^y(S^n_{xy}) + \delta^z(S^n_{xz})\ \right]$$

$$v^{n+1} = v^n + \frac{\Delta t}{\rho^n}\left[\ \delta^y(S^n_{yy} - p^{n+1/2}) + \delta^x(S^n_{xy}) + \delta^z(S^n_{yz})\ \right]$$

$$w^{n+1} = w^n + \frac{\Delta t}{\rho^n}\left[\ \delta^z(-S^n_{xx} - S^n_{yy} - p^{n+1/2}) + \delta^x(S^n_{xz}) + \delta^y(S^n_{yz})\ \right]$$

where:

$$p^{n+1/2}_{i+1/2} = p^n_{i+1/2} - \frac{\Delta t}{2}\,\rho C^2_{i+1/2}\,\delta^x(u^n_{i+1/2})$$

and:

$$p^n_{i+1/2} = \frac{\rho_{i+1}p^n_i + \rho_i p^n_{i+1}}{\rho_i + \rho_{i+1}}$$

$$\rho C^2_{i+1/2} = \text{MIN} \quad (\rho C^2_{i,j,k} \quad , \rho C^2_{i+1,j,k})$$

$$u^n_{i+1/2} = \frac{\rho_i u^n_i + \rho_{i+1} u^n_{i+1}}{\rho_i + \rho_{i+1}}$$

Solution of the conservation of energy equation is:

$$E^{n+1} = E^n + \frac{\Delta t}{\rho^n} \Big[\delta^x \left(-P\, u^{n+1/2} + S^n_{xx}\, u^{n+1/2} + S^n_{xy}\, v^n + S^n_{xz}\, w^n\right)$$

$$+ \quad \delta^y \left(-P v^{n+1/2} + S^n_{yy}\, v^{n+1/2} + S^n_{xy}\, u^n + S^n_{yz}\, w^n\right)$$

$$+ \quad \delta^z \left(-P w^{n+1/2} + S^n_{zz}\, w^{n+1/2} + S^n_{xz}\, u^n + S^n_{yz}\, v^n\right) \Big]$$

where the boundary value velocities at time n+1/2 are given by:

$$u^{n+1/2}_{i+1/2} = u^n_{i+1/2} - \frac{\Delta t}{2\rho^{n+1/2}_{i+1/2}} \quad \delta^x (P^n_{i+1/2})$$

with similar forms for v and w with

$$u^n_{i+1/2} = \frac{\rho_i u^n_i + \rho_{i+1}\, u^n_{i+1}}{\rho_i + \rho_{i+1}}$$

$$\rho^{n+1/2}_{i+1/2} = \rho^n_{i+1/2} \quad \left(1 - \frac{\Delta t}{2} \delta^x\, u^n_{i+1/2}\right)$$

$$\rho^n_{i+1/2} = \text{Maximum } (\rho^n_i \, , \, \rho^n_{i+1}).$$

Each of the $S_{\alpha\beta}$ terms are calculated in a manner similar to S_{xx} as:

$$S_{xx_{i+1/2}} = \phi_{i+1/2} \, \frac{\rho_{i+1}\, S_{xx_i} + \rho_i\, S_{xx_{i+1}}}{\rho_i + \rho_{i+1}} \, ,$$

$$\phi_{i+1/2} = \text{MIN}(vf_i, \, vf_{i+1})$$

and vf_i = the fraction of zone i, j, k which is occupied by a solid material. The time n+1 deviators are calculated by:

$$S_{xx}^{n+1} = S_{xx}^{n} + 2G(\delta^x(u^{n+1/2}) - D)\Delta t$$

$$S_{yy}^{n+1} = S_{yy}^{n} + 2G(\delta^y(v^{n+1/2}) - D)\Delta t$$

$$S_{xy}^{n+1} = S_{xy}^{n} + G(\delta^x(v^n) + \delta^y(u^n))\Delta t$$

$$S_{xz}^{n+1} = S_{xz}^{n} + G(\delta^x(w^n) + \delta^z(u^n))\Delta t$$

$$S_{yz}^{n+1} = S_{yz}^{n} + G(\delta^y(w^n) + \delta^z(v^n))\Delta t$$

where:

$$D = [\delta^x(u^{n+1/2}) + \delta^y(v^{n+1/2}) + \delta^z(w^{n+1/2})]/3.$$

The stresses are then compared with the yield surface definition by:

$$J = S_{xx}^2 + S_{yy}^2 + S_{xx}\,S_{yy} + S_{xy}^2 + S_{xz}^2 + S_{yz}^2 \leq Y^2/3$$

If this criterion is not met, then each stress component is reset by:

$$S_{\alpha\beta} = \left[\frac{Y^2}{3J}\right]^{1/2} S_{\alpha\beta}$$

where $S_{\alpha\beta}$ represents each of the elements of the stress tensor.

ADVECTION PHASE CALCULATION

Up to this point the entire calculation has been in a Lagrangian framework. The next step is to indicate the volume change due to spherical forces or pressure and prepare the system for the fluxing calculation. This is done by solving the continuity equation by:

$$V^{n+1} = V^n(1+3\Delta t D)$$

where V is the cell volume. The equation of state is then called and the pressure in the new zone is calculated. If the zone contains more than one material, an iteration procedure is used to equilibrate the pressure of each material species in the zone to a common value. This is done by repartitioning the zone among the materials present. Species energy is calculated by applying $\Delta t = P\ V$ as the volume V occupied by each species at pressure P subject to conserving the total mass, volume, and internal energy of the zone.

The calculation is returned to a fixed (or uniformly translating or expanding) grid by reapportioning material from the displaced to the fixed grid. The volume to be fluxed is defined as the sum of the transfer volumes in each of the three directions, which are given by:

$$V_{i+1/2} = (x^{n+1}_{i+1/2} - x^{n}_{i+1/2})\ \Delta y_j\ \Delta z_k\ V^{n+1}/V^n \qquad \text{(A-35)}$$

$$V_{j+1/2} = (y^{n+1}_{j+1/2} - y^{n}_{j+1/2})\ \Delta x_i\ \Delta z_k\ V^{n+1}/V^n \qquad \text{(A-36)}$$

$$V_{k+1/2} = (z^{n+1}_{k+1/2} - z^{n}_{k+1/2})\ \Delta x_i\ \Delta y_j\ V^{n+1}/V^n \qquad \text{(A-37)}$$

where

$V_{i+1/2}$ = volume (in cm^3) to be transported between cell i,j,k and cell i+1,j,k

$$x^{n+1}_{i+1/2} = x^{n}_{i+1/2} + u^{n+1}_{i+1/2}\ \Delta t$$

$$x^{n}_{i+1/2} = \frac{x^{n}_{i} + x^{n}_{i+1}}{2}$$

$$u^{n+1}_{i+1/2} = \frac{u^{n+1}_{i}\ \rho^{n}_{i} + u^{n+1}_{i+1}\ \rho^{n}_{i+1}}{\rho^{n}_{i} + \rho^{n}_{i+1}}$$

with analogous definitions for j and k.

The transfer volume defined by Equations (A-35) - (A-37) is apportioned among the available materials in the donor cell in accordance with the diffusion limiter algorithm. This algorithm is ad hoc but has been formulated

over the years by observation of various numerical experiments and has been found to be modestly successful. The algorithm uses nearest neighbor information in an attempt to unmix mixed material cells. This is done by assigning a relative transport weight to each material in the donor cell, and then normalizing to the transfer volume given in Equations (A-35) - (A-37). Each direction (up to 6 in 3D) is handled separately subject to the constraint that the sum of the exiting volumes for each material shall not exceed the initial volume of that material.

The relative transport weight W_n for each material n is defined by

$$W_n = \left[\frac{V_{nR} - V_{nU}}{V_{nR} + V_{nU}} + 1.0 \right]^2$$

$$= \left[\frac{2V_{nR}}{V_{nR} + V_{nU}} \right]^2 \qquad \text{(A-38)}$$

where V_{nR} = fractional volume of material n in the receiver cell

V_{nU} = fractional volume of material n in the upstream cell.

The fractional volume is defined as the volume of material n divided by the total cell volume. Two exceptions to Equation (A-38) are noted. If there is no material n in the donor cell then $W_n = 0$, and if V_{nR} and V_{nU} are zero then $W_n = 1$.

Finally the transport fraction, Δ_{nm}, for each material n in a particular direction m is obtained by normalizing the relative transport weights by

$$\Delta_{nm} = \frac{V_F}{\sum_n W_n V_n} W_n$$

where V_F = transfer volume given by Equation (A-35),(A-36), or (A-37)

V_n = volume of material n in donor cell.

The transport fractions Δ_{nm} are subject to the constraint that the sum over

all directions (six in 3D) shall not exceed unity for each material n. If it does, suitable corrections are made.

The transport fraction applies not only to the volume but also to the mass and internal energy for material n. That is, the fraction Δ_{nm} of each of these quantities will be transferred from the donor to the receiver cell. The sum over n for each direction defines the total flux of mass and internal energy.

The transport methodology will be illustrated for one direction (flux to the right). The remaining directions are analogous. Consider cell i,j,k at the end of the Lagrangian phase, denoted above as time n+1. For each material n the amount of mass transporting is

$$\delta M_n^R = (\Delta_{nR}\, M_n)_{donor}$$

while the volume transporting is

$$\delta V_n^R = (\Delta_{nR}\, V_n)_{donor}$$

and the internal energy is

$$\delta (MI)_n^R = [\Delta_{nR} (MI)_n]_{donor}.$$

The total mass fluxing (to the right) is therefore

$$\delta M^R = \sum_n \delta M_n^R$$

while the internal energy is

$$\delta (MI)^R = \sum_n \delta (M_n I_n)^R.$$

The total energy is

$$\delta (ME)^R = \delta (MI)^R + 1/2\ \delta M^R\ (u_d^2 + v_d^2 + w_d^2)$$

where d indicates donor cell velocities. The transporting material carries with it a proportionate amount of momentum, thus

$$\delta(Mu)^R = u_d \delta M^R$$

$$\delta(Mv)^R = v_d \delta M^R$$

$$\delta(Mw)^R = w_d \delta M^R$$

With most options HULL transports certain histories from cell to cell. These histories essentially transport with the mass; hence, for the kth history H

$$\delta H_k^R = H_{dk} \delta M^R$$

where d again indicates the donor cell.

Similar terms are obtained for the other directions fore (F) and above (A), and from previous calculations for left (L), aft (T), and below (B).

Terms such as the mass $M_{i,j,k}^{n+1}$ are redefined from the displaced

Lagrangian mesh to the original fixed (or uniformly translating) Eulerian mesh $e_{M_{i,j,k}^{n+1}}$ as follows. For material n

$$e_{M_n^{n+1}} = M_n^{n+1} + \delta M_n^L + \delta M_n^T + \delta M_n^B - \delta M_n^R - \delta M_n^F - \delta M_n^A$$

$$e_{V_n^{n+1}} = V_n^{n+1} + \delta V_n^L + \delta V_n^T + \delta V_n^B - \delta V_n^R - \delta V_n^F - \delta V_n^A$$

$$e_{(MI)_n^{n+1}} = (MI)_n^{n+1} + \delta(MI)_n^L + \delta(MI)_n^T + \delta(MI)_n^B - \delta(MI)_n^R - \delta(MI)_n^F - \delta(MI)_n^A$$

The velocity is defined by conserving momentum with

$$e_u{}^{n+1} = [u^{n+1} M^{n+1} + \delta(Mu)^L + \delta(Mu)^T + \delta(Mu)^B - \delta(Mu)^R - \delta(Mu)^F - \delta(Mu)^A] / \sum_n e_{M_n^{n+1}}$$

$$e_v{}^{n+1} = [v^{n+1} M^{n+1} + \delta(Mv)^L + \delta(Mv)^T + \delta(Mv)^B - \delta(Mv)^R$$

$$-\delta(Mv)^F - \delta(Mv)^A] \Big/ \sum_n e_{M_n}^{n+1}$$

$$e_w^{n+1} = [w^{n+1} M^{n+1} + \delta(Mw)^L + \delta(Mw)^T + \delta(Mw)^B - \delta(Mw)^R$$
$$-\delta(Mw)^F - \delta(Mw)^A] \Big/ \sum_n e_{M_n}^{n+1}$$

The new internal energy is

$$e_I^{n+1} = [I^{n+1} M^{n+1} + \delta(ME)^L + \delta(ME)^T + \delta(ME)^B - \delta(ME)^R - \delta(ME)^F - \delta(ME)^A]$$
$$\Big/ \sum_n e_{M_n}^{n+1} - 1/2\ [(e_u^{n+1})^2 + (e_v^{n+1})^2 + (e_w^{n+1})^2]$$

The new histories are defined by

$$e_{H_k}^{n+1} = (H_k^{n+1} M^{n+1} + \delta H_k^L + \delta H_k^T + \delta H_k^B - \delta H_k^R - \delta H_k^F - \delta H_k^A) \Big/ \sum_n e_{M_n}^{n+1}$$

It should be noted that, in general,

$$(e_M^{n+1})(e_I^{n+1}) \neq \sum_n e_{(MI)_n}^{n+1}$$

as these terms are defined above. This is due to a dissipation of kinetic into internal energy. Total energy is conserved by adjusting the individual energies.

APPENDIX B

EQUATIONS-OF-STATE

The equations-of-state do not generally attempt to accurately cover all regions of P, V, I space. They were developed to provide valid material response in problems of interest in conventional munitions research - shaped charge jet formation, self forging fragment formation, metal-on-metal penetrations, bomb penetrations into concrete, and other similar problems. This emphasis has meant, for example, that great care was given to modelling solid and liquid metal behavior, but metal vapor behavior was simpl fied to an ideal gas formulation. Problems of conventional munitions research interest do not usually require highly accurate metal vapor response.

The HULL equations-of-state (EOS 6) have been implemented in EPIC3 with some minor simplifications. Thus, this report can serve as documentation for many of the EPIC3 equations-of-state as well as those in HULL.

In what follows, the term 'equations-of-state' will refer to the determination of all stress-strain relations including deviatoric as well as hydrostatic behavior.

1. Metal Equations-of-State

The metal equation-of-state relationships are based on Mie-Gruneisen and gamma-law gas pressure formulations and the von Mises yield criterion. The yield strength can be a bi-linear function of accumulated plastic strain (work hardening) and a tri-linear function of internal energy (thermal softening). The internal energy density (energy per unit mass) at onset of melt and vaporization are increased with increasing density. Increasing internal energy is not allowed to increase pressure during the melting and vaporization phase changes. We have made no provision for solid-solid phase changes although such phase changes could be added relatively simply.

A. The Pressure Equations

The basic HULL pressure equation for a solid or liquid metal is the Mie-Gruneisen equation:

$$P = P_H (1-\Gamma\mu/2) + \rho(I-I_0) + P_0$$

where:

P_H is the Hugoniot pressure given by:

$$P_H = B_1\mu + B_2\mu^2 + B_3\mu^3 \quad \text{if } \mu \geq 0$$

$$= B_1\mu \quad \text{if } \mu \leq 0$$

and μ is the excess compression:

$$\mu = \rho/\rho_0 - 1$$

ρ is current density

ρ_0 is ambient density

Γ is the Gruneisen parameter $(V (\frac{\partial P}{\partial I})_V)$ given by:

$$\Gamma = \Gamma_0\rho_0/\rho$$

I is the internal energy per unit mass

I_0 is the internal energy per unit mass at ambient density and pressure (1 atmosphere)

P_0 is ambient pressure and is calculated from

$$P_0 = \left(\frac{1.013 \times 10^6 \text{ dynes/cm}^2}{I_0}\right). \text{ Min } (I, I_0)$$

This allows P_0 to be 1 atmosphere when $I = I_0$ and to decrease smoothly to zero as I drops below the ambient level (I_0).

The Hugoniot coefficients are calculated from:

$$B_1 = \rho_o C_o^2$$

$$B_2 = B_1 \ [1 + 2 \ (s-1)]$$

$$B_3 = B_1 \ [2 \ (s-1) + 3 \ (s-1)^2]$$

where C_o and s define the linear shock velocity (U_s) versus particle velocity (U_p) relationship.

The assumption of a linear U_s versus U_p relationship:

$$U_s = C_o + s \cdot U_p$$

combined with the Hugoniot jump conditions actually results in the Hugoniot pressure equation:

$$P_H = \rho_o C_o^2 \ \mu(\mu + 1)/[1 - \mu(s-1)]^2$$

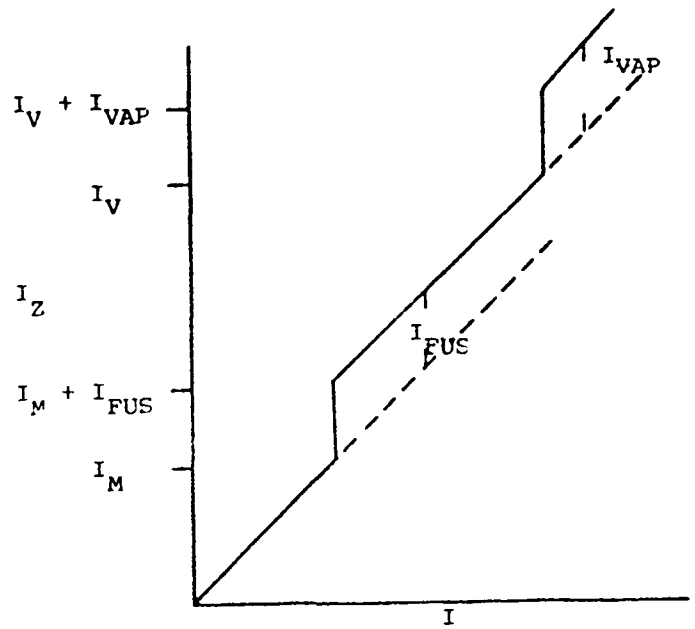
Expanding the denominator by the binomial theorem, simplifying and deleting all terms with the multipliers beyond μ^3 yields the coeffients B_1, B_2 and B_3. This expansion process has also deleted the pole in the P_H formulation.

The Gruneisen parameter assumption, namely

$$\Gamma = \Gamma_o \rho_o / \rho$$

is considered reasonable for metals (Reference 1) and results in well-behaved pressures at high values of density. The other often used assumption that does not change with increasing density results in pressure maxima for relatively small density increases.

The internal energy density, I, used in the Mie-Gruneisen pressure calculation is not always the zone's internal energy density, I_z. The relationship is seen below:



where I_M is the internal energy density at the onset of melt

I_{FUS} is the energy required to complete fusion or the melting process

I_V is the internal energy density at the onset of vaporization

I_{VAP} is the energy required to complete vaporization.

As the diagram indicates, I is equal to I_z minus internal energy lost in the fusion and/or vaporization process. Thus pressure due solely to energy addition is not allowed to increase during fusion and vaporization until these processes are complete. Density increases will still provide pressure increases through the first term in the equation.

Metal energy at the onset of melt increases with increasing confinement. Thus, the I_M and I_V should be changed as pressure increases. HULL uses the GRAY metal (Reference 6) fit for I_M as a function of volumetric strain. The function is:

$$I_M = I_{MO} (1 + a\eta + b\eta^2)$$

where:

$$a = 2\,\Gamma_o - 2/3$$

$$b = (\Gamma_o - 1/3)\,(2\,\Gamma_o + 1/3) - 1$$

$$\eta = 1 - \rho_o/\rho$$

I_{MO} = internal energy density at onset of melt at ambient density

We increase I_V using the same function, i.e.,

$$I_V = \frac{I_M}{I_{MO}}\ I_{VO}$$

I_{FUS} and I_{VAP} are constant regardless of the zone's compression.

The HULL equation-of-state transitions to a gamma-law gas at internal energies above sublimation energy and whenever the pressure so computed is higher than the Mie-Gruneisen pressure. The pressure comparison ensures a smooth transition between equations without actually determining and identifying intersections of the two regions in P, V, I space. The gamma-law equation is:

$$P = (\gamma - 1)\ \rho\ (I - I_s)$$

where I_s is the internal energy density for sublimation and γ is C_p/C_v. The I in this equation is the zone's internal energy--i.e., it is not reduced by I_{FUS} or I_{VAP}.

There are a number of constraints applied to the Mie-Gruneisen pressure to more physically model metal behavior in extreme energy and density states. The constraints are more easily explained by expressing the Mie-Gruneisen equation as:

$$P = F_1\ (\rho) + F_2\ (\rho) \cdot (I - I_o) + P_o$$

When material in a zone expands under tension, it should not be allowed to do so indefinitely. Use of the void insertion fracture model in HULL can prevent extreme expansion states from occurring. However, this model requires data which is often unavailable. In these cases the following

constraints will provide some physical realism.

The product $F_2(I - I_o)$ is limited if μ is less than -0.25. The equation employed is:

$$F_2 (I - I_o) = [F_2 (I - I_o)] \, 20 \, (\mu + 0.4)/3$$

where the term in brackets is the product calculated without constraints. The equation provides a linear descent to a value of zero for the product at ; = -0.4. Because of an IF check, the equation is not invoked unless ; is less than -0.25. The point of controlling this product is simply that metals at such high expansions cannot support compression. The zone contains metal fragments, and such fragments cannot support compression when they are so highly distended. Without this control, compressive pressures could still exist with high enough I values.

The Gruneisen parameter is limited so as not to exceed $2\Gamma_o$. In highly expanded states the Gruneisen parameter grows without limit as density decreases. Since the Gruneisen parameter is not meant to be applied in expanded zones, it is reasonable to control this growth. This control is somewhat redundant with the $F_2(I-I_o)$ control.

If the calculated Mie-Gruneisen pressure is tensile, there are several controls which are applied.

If the pressure is less than P_{MIN} $(1-0.1\mu)$, it is reset to P_{MIN} $(1-0.1\mu)$. Normally P_{MIN} is set to a very large negative number (-1×10^{20}) so that this control is not invoked. However, the same equation of state routines are used for media such as water which can support no tension. In such cases, this control is required. The factor $(1-0.1\mu)$ provides a variable pressure as μ changes. The product P_{MIN} $(1-0.1\mu)$ grows slightly more negative as μ decreases.

If the Mie-Gruneisen pressure is tensile and μ is less than -0.2, the pressure is reset to:

$$-20 \, (\mu + 0.2) \, (80 - P) + P$$

where P is the pressure after the P_{MIN} chec If μ is less than -0.25, the pressure is reset to:

$$100\ (1 + \mu)$$

These two controls work together to limit total pressure to very small positive values along a linear ramp from P = 0 at $\mu = -1$ to P = 100 dynes/cm^2 at $\mu = 0$. This provides essentially zero pressures with a small positive slope as density increases.

The metal's tensile capability is limited as its internal energy density approaches melt energy. To use this control, the quantity:

$$I_{MY} = I_M + 0.5\ I_{FUS}$$

is calculated. This is also the quantity used to limit yield strength and will be discussed later in this section. If I is greater than 0.6 I_{MY}, the calculated tensile Mie-Gruneisen pressure is multiplied by the factor:

$$F = \frac{2.5}{(I_{MY}-I_o)}(I_{MY}-I) \quad \text{if } I_{MY} > 0.61_{MY}$$

$$= 0 \qquad \text{if } I > I_{MY}$$

and then reset to

$$F \cdot P + 100\ (1 + \mu)$$

This control limits tensile values of pressure as I increases to I_{MY} and phases the pressure into the distended fragment-filled zone curve previously discussed.

Compressive Mie-Gruneisen pressures are limited to the region of 20 x 10^{12} dynes/cm^2 (20 megabars). A real material cannot achieve such pressures without also having internal energies considerably greater than I_s. In this case, the equation of state will have switched to a gamma-law gas which has no compressive controls on it. Therefore, if a call is made to the equation of state where P exceeds 20 megabars, it is reset to:

$$P = 2 \times 10^{11} + 2 \times 10^{11} (\mu-\mu_{20})/(100 - \mu_{20})$$

where μ_{20} is the value of μ when P is 2×10^{11}. This provides a gentle slope with density for values of μ greater than μ_{20}.

Before adjusting yield strength, the equation-of-state routines provide a few more variables required in other sections of HULL.

The mixed-zone equation-of-state iterator (Reference 5) requires $(\frac{\partial P}{\partial \tau})_I$, where τ is the specific volume, $1/\rho$. HULL calculates $(\frac{\partial P}{\partial \rho})_I$ and multiplies by $-\rho^2$ to obtain this quantity.

The adiabatic bulk sound speed is required for several purposes, including time step control. In the absence of heat conduction, the adiabatic sound speed is the isentropic sound speed, C_s, calculated from the thermodynamic identity (Reference 1):

$$C_s = (\frac{\partial P}{\partial \rho})_S = (\frac{\partial P}{\partial \rho})_I + \frac{P}{\rho^2}(\frac{\partial P}{\partial I})_\rho$$

For both pressure derivatives HULL takes account of the controls invoked and computes derivatives along appropriate paths.

An approximate temperature is calculated, for edit purposes only, from the equation:

$$T(^\circ K) = I/C_v + T_o$$

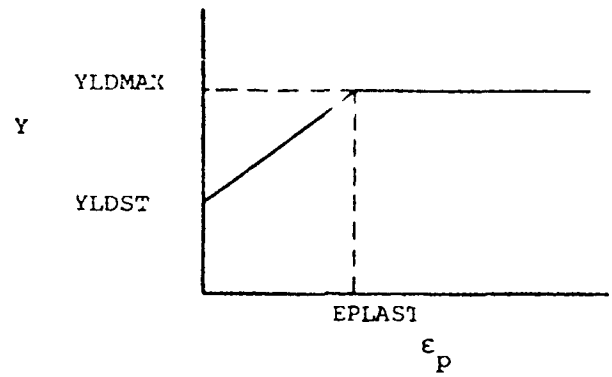
where C_v is calculated from:

$C_v = 3R/AW$
R = gas constant = 8.134×10^7
AW = the metal's atomic weight

and where T_o is the temperature required to force T to a value of 288°K when I is I_o.

B. Yield Strength Equations

If the WORK option is set to 1 in the KEEL (generator) input deck, yield strength is varied as a function of accumulated plastic strain in a form as seen below.



where YLDST is the initial (zero plastic strain) yield strength and YLDMAX is the ultimate yield strength which occurs when ε_p = EPLAST. If WORK = 0, the ultimate yield strength is used for all values of ε_p.

The calculation of ε_p, incremental strains and stresses, and resetting of deviatoric stresses exceeding the yield limit are carried out in the usual fashion and completely described in the previous section. These calculations will not be repeated here.

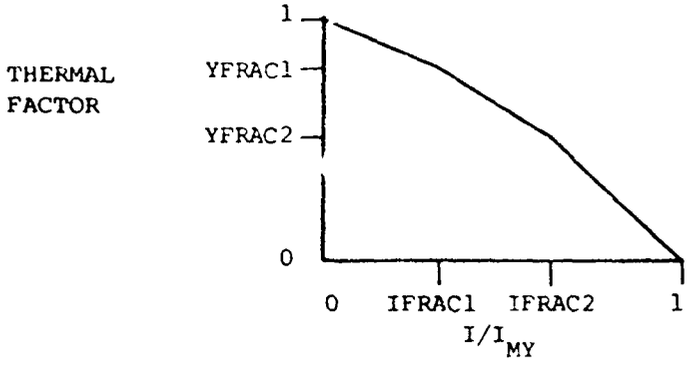
After an ambient energy (possibly work hardened) yield strength is calculated, it is multiplied by a thermal softening factor which varies from 1 at ambient energy to 0 at I_{MY}. I_{MY} is the melt energy at which all yield strength disappears. HULL sets this value to:

$$I_{MY} = I_M + 0.5I_{FUS}$$

where I_M is the onset of melt energy increased as a result of increased density and I_{FUS} is the energy absorbed during the melting process. This is a somewhat arbitrary assignment. There will be some point during the fusion process when enough centers of melt exist to drive yield strength to essentially zero values. The exact position of this point is unknown and will probably never be known since observables such as pressure and temperature do not vary. It does not seem that the metal would have to be entirely melted to

essentially zero any strength characteristics. On the other hand, when a few very small centers of melt are being created—i.e., at the onset of melt—it does not seem that the strength of the metal would be appreciably affected. To avoid both extremes, HULL uses the midpoint of the fusion process.

The thermal softening factor is determined from a trilinear curve as seen below:



where the pairs (YFRAC1,IFRAC1), (YFRAC2,IFRAC2) are material constants.

HULL employs the ratio I/I_{MY} instead of temperature to calculate a thermal softening factor simply because the relationship between temperature and internal energy is virtually unknown at densities other than ambient. The HULL technique, in effect, implies that there is a constant relationship between I/I_{MY} and T/T_M at any given density but does not attempt to determine the value of the constant.

It is possible that a highly expanded zone could have a significant value of yield strength, when, in reality, the rubble in the zone would be too distended to support any strength. To prevent this, yield strength is multiplied by another factor in addition to work hardening and thermal softening. This factor is 1 for μ values greater than -0.2. It varies linearly from 1 at $\mu = -0.2$ to 0 at $\mu = -0.25$ and then is 0 for all values of μ less than -0.25.

HULL does not calculate an explicit multiplier for strain rate variations of yield strength. Metals demonstrate the property of having an almost constant yield strength as strain rates vary from 10^3 sec^{-1} to 10^5 sec^{-1}. Since conventioral weapon calculations lie almost completely within this

range, HULL assumes that the strength values used will be appropriate within this range.

HULL has built-in values for all metal material properties. Strength properties can be changed through use of the MATPROP option in the HULL input deck. Other property changes require changing DATA statements in the code. Tables B-1 and B-2 summarize the built-in property values for materials employing the Mie-Gruneisen equation-of-state. The Hugoniot data was obtained from References 1, 10, and 11 and is considered very accurate. Melt and vaporization energy values were obtained from many sources and considered reasonably accurate unless noted otherwise in the tables. Yield strength values and thermal softening data were also obtained from many sources including material property tests at the U.S. Air Force Materials Laboratory, the fitting of calculational results to ambient and elevated temperature cylinder impact tests, and the fitting of calculational results to self forging fragment formation experiments. However, users should consider all of the strength data as suspect and attempt to obtain data valid for the particular metal alloy of concern. When such data has been found or generated, the user can override built-in strength properties by entering the following cards in HULL input:

```
MATPROP    MAT    = N
           YLDST  = Y1        YLDMAX = Y2        EPLAST = E
           YFRAC1 = YF1       IFRAC1 = IF1
           YFRAC2 = YF2       IFRAC2 = IF2
ENDPROP
```

where Y_1, Y_2, E, YF_1, IF_1, YF_2, and IF_2 are new input values, and N is the material number in the HULL calculation. The example illustrates all stress surface values being changed. Not all values have to be changed, and more than one material can be altered.

2. Plain Concrete and Sand Equations of State

HULL contains a plain concrete equation-of-state model developed to provide reasonable response in the 1- to 300-kilobar range. It is based on a very

TABLE B-1. HULL MATERIAL PROPERTIES

MATERIAL	HULL NAME	ρ_o (gm/cc)	C_o (10^5 cm/sec)	s	Γ_o	PMIN (dynes/cm^2)
Aluminum	AL	2.71	5.38	1.337	2.1	$-1.x10^{20}$
Ceramic (Al_2O_3)	CERAM	3.9	6.9	1.45	0.5	$-1.x10^{20}$
Copper	CU	8.9	3.958	1.497	2.0	$-1.x10^{20}$
Iron	FE	7.86	4.61	1.73	1.67	$-1.x10^{20}$
Lead	PB	11.34	2.092	1.452	2.0	$-1.x10^{20}$
Nickel	NI	8.9	4.8	1.254	1.83	$-1.X10^{20}$
Nylon	NYLON	1.14	2.29	1.63	0.87	$-10x10^{9}$
Plastic (polyrubber)	PLAST	1.012	0.854	1.865	0.9	$-10x10^{9}$
RHA (thin plate)	RHA	7.86	4.61	1.73	1.67	$-1.x10^{20}$
Stainless Steel	SSTEEL	SAME PROPERTIES AS IRON				
Tantalum	TA	16.6	3.423	1.214	1.69	$-1.x10^{20}$
Teflon®	TEFLON	2.16	1.34	1.93	0.9	$-10x10^{9}$
Tungsten	W	18.1	4.0	1.268	1.58	$-1.x10^{20}$
Uranium	U	19.05	2.48	1.53	1.6	$-1.x10^{20}$
U-0.75%TI	UTI	18.9	2.48	1.53	1.6	$-1.x10^{20}$
Water	WATER	1.0	1.483	1.75	0.28	0.0

TABLE B-1. HULL MATERIAL PROPERTIES (CONCLUDED)

MATERIAL	I_{MO} (10^9ergs/gm)	I_{FUS} (10^9ergs/gm)	I_{VO} (10^9ergs/gm)	$I_{VO}+I_{VAP}$ (10^9ergs/gm)	I_s (10^9ergs/gm)	$\gamma-1$(vapor equation)
Aluminum	7.3	3.96	32.0	100.0*	119.2	0.25
Ceramic (Al_2O_3)	9.55	2.1	50*	100.0*	100.0*	0.25
Copper	4.78	2.12	16.3	63.0	53.1	0.25
Iron	7.4	2.74	22.4	86.8	74.2	0.25
Lead	0.67	0.262	2.96	11.2	9.4	0.25
Nickel	6.6	3.1	20.1	84.7	72.9	0.25
Nylon	1.9	2.1	50.0*	100.0*	100.0*	0.25
Plastic (polyrubber)	20.0*	2.0*	50.0*	100.0*	100.0*	0.25
RHA (thin plate)	7.4	2.74	22.4	86.8	74.2	0.25
Stainless Steel	SAME PROPERTIES AS IRON					
Tantalum	4.32	1.59	12.5	48 3	43.2	0.25
Teflon®	20.0*	2.1*	50.0*	100.0*	100.0*	0.25
Tungsten	4.77	1.84	13.6	58.4	46.0	0.2
Uranium	1.4	0.49	5.5	28.2	22.7	0.25
U-0.75%TI	1.4	0.49	5.5	28.2	22.7	0.25
Water	7.1	3.35	23.8	44.7	44.7	0.25

*NO DATA, ARBITRARY VALUES ASSIGNED.

TABLE B-2. HULL STRENGTH PROPERTIES

MATERIAL	YLDST (10^9 DYNES/CM^2)	WYLDST (10^9 DYNES/CM^2)	EPLAST	YF1	IF1	IF2	EF2	SHEAR MODULUS (10^{11} DYNES/CM^2)
Aluminum	2.9	2.9	0.0	0.9	0.5	0.9	0.5	2.69
Ceramic (Al_2O_3)	80.0	80.0	0.0	0.21	0.45	0.05	0.75	10.0
Copper	1.2	4.5	0.3	0.78	0.41	0.78	0.41	4.64
Iron	4.69	5.5	0.3	0.9	0.5	0.9	0.5	6.41
Lead	0.3	0.3	0.0	0.9	0.8	0.9	0.8	1.113
Nickel	6.0	6.0	0.0	0.8	0.5	0.8	0.5	8.9
Nylon	0.5	0.5	0.0	0.9	0.5	0.9	0.5	0.368
Plastic (Polyrubber)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
RHA	15.0	15.0	0.0	0.9	0.5	0.9	0.5	6.41
Stainless Steel	6.89	10.0	0.3	0.9	0.5	0.9	0.5	7.3
Tantalum	5	5	0.0	0.5	0.4	0.5	0.4	6.56
Teflon®	0.5	0.5	0.0	0.9	0.5	0.9	0.5	0.233
Tungsten	20.0	20.0	0.0	0.9	0.5	0.9	0.5	16.0
Uranium	8	15	0.5	0.9	0.5	0.9	0.5	5.1
U-0.75%TI	8	15	0.5	0.9	0.5	0.9	0.5	5.1
Water	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

small amount of data, since most available data is at stress levels well below 1 kilobar. However, the equation-of-state has been used successfully in calculations of penetrations into concrete at velocities from a few hundred to over 1,000 feet/second (Reference 12).

The model was constructed primarily from Hugoniot data on one specific concrete (Reference 13) and static yield strength data on several concretes (Reference 14). Assumptions were made for a value for Poisson's ratio, a shear modulus formula, a plastic flow rule, and other parameters.

Gregson (Reference 13) generated Hugoniot data to stress levels over 500 kilobars for a granite aggregate concrete. The aggregate had an average diameter of 1/8 inch. The initial density varied from mix to mix but averaged 2.185 gm/cc. The solid density had an average of 2.673 gm/cc. The concrete consisted of 18 percent voids, 25 percent granite aggregate, 20 to 25 percent quartz grains, and 25 to 30 percent cement paste by volume. The small aggregate size was dictated by his shock-generating method, which consisted of gas-gun launched flyer plates. He employed several measurement techniques to establish stress and velocity. The final data was converted to stress versus excess compression, μ.

The lowest point in Gregson's data is at 0.75 kilobar. This is a fairly consistent precursor value seen in his stress versus time data. HULL uses that point to define the density ratio at which significant cracking and void filling occurs.

The estimated unloading path is simply one which allows the concrete to return to its solid density when loaded past the point where all voids are filled. It is a straight line which connects with the loading curve at the total void filled point with a slope equal to the loading curve at that point. There is no data for this line beyond the solid density point at zero stress.

The yield strength of concrete increases as confining pressure is increased. The only data available at high confining pressures is static data generated by Chinn (Reference 14). Chinn's results were used along with some

typical low stress level results from McHenry (Reference 15). The yield strength and pressure are normalized by f'_c, the concrete's unconfined compressive strength. Two theoretical points were chosen. One defines failure (flow) at the unconfined compressive strength point, i.e., $Y = f'_c$ at $P = 1/3\ f'_c$. The other is included because a minimum pressure criterion is used to define failure. This minimum pressure was chosen as that occurring in an unconfined tensile test assuming failure when stress is $0.1\ f'_c$ (Reference 16). It is numerically pleasing to set $Y = 0$ at this failure point.

The yield strength curves are approximated in HULL with three straight line segments:

$$Y = 0 \qquad \text{if } P \leq -0.1f'_c$$

$$Y = 3\ (P + 0.1f'_c/3)/1.1 \qquad \text{if } -0.1f'_c/3 \leq P \leq f'_c/3$$

$$Y = P + 2/3\ f'_c \qquad \text{if } f'_c/3 < P \leq 30f'_c$$

$$Y = 30.67\ f'_c \qquad \text{if } P > 30\ f'_c$$

the limit at $P = 30f'_c$ is not indicated in Chinn's data. It is included because it is felt there should be a limiting value. It is included at the level shown simply because it is not known to exist at a lower level.

It should be emphasized that the yield strength data was developed in static tests. Higher yield strengths may exist at strain rates of interest in conventional munitions work, but data simply does not exist.

Using the previous equations for Y, Gregson's Hugoniot stress data was reduced to provide an Hugoniot pressure versus excess compression curve by simply subtracting 2/3 Y from the stress data assuming an f'_c value of 5,000 PSI.

Below the precursor pressure level of 0.358 kilobar (1/3 x 0.75 kilobar), Hugoniot pressure is fit with a straight line:

$$P_H = K_0\mu$$

where K_0 is determined from the empirical Young's modulus relationship (Reference 10)

$$E = 33W^{1.5} (f'_c)^{0.5} PSI \text{ (W in lb/ft}^3 \text{ = density)}$$

and the relationship

$$K = E/3(1-2\sigma)$$

with a value of Poisson's ratio, σ, chosen as 0.2.

Designating the value of μ at 0.358 kilobar as μ_E, the loading Hugoniot pressure was fit with the following equations:

P_H(Kb)	RANGE OF μ
$K_0\mu$	-0 to μ_E
$0.358 + 78.62 (\mu-\mu_E)$	μ_E to 0.1
$8 + 130 (\mu - 0.1)$	0.1 to 0.2
$21 + 420 (\mu - 0.2)$	0.2 to 0.3
$63 + 650 (\mu - 0.3)$	0.3 to 0.366 (lockup point)
$76 + 625.5 (\mu - 0.32) + 1720.19 (\mu - 0.32)^2$	0.366 to 0.414
$150 + 39.68 (\mu - 0.414)$	0.414 to 0.54 (solid - solid phase transition)
$166 + 360.5 (\mu - 0.572) + 864 (\mu - 0.572)^2$	0.54 to ∞

The maximum density unloading curve was fit by the equation:

$$P_H = 784 (\mu - 0.223)$$

HULL maintains a zone variable, μ_M, for concrete which defines the maximum value of μ previously seen by the zone. For values of μ_M less than μ_E, loading, unloading and reloading all occur along the same elastic path. For values of μ_M greater than μ at lockup (0.366) loading, unloading, and reloading take place along the same path. For μ_M values between these extremes, unloading and reloading up to the virgin loading curve take place along K_μ lines where K is chosen to vary linearly between K_o and 784 kilobars.

The shear modulus is calculated assuming that Hooke's law holds. That is:

$$G = \frac{3(1-2\sigma)}{2(1+\sigma)} \cdot K_b$$

where K_b is the bulk modulus, $\rho\ (\frac{\partial P}{\partial \rho})$, and σ is Poisson's ratio.

Deviators are reset as though the yield surface were a von Mises surface--i.e., not a function of pressure. The resulting plastic flow rule is inconsistent with the Mohr-Coulomb yield surface proposed for the concrete. However, the alternative, such as a moving CAP model (Reference 2), is computationally complex and cannot be justified considering the source of the yield strength model. Strain rate effects are probably far more important than a consistent flow rule.

The Hugoniot pressure previously discussed could be combined with a Gruneisen parameter into a complete equation of state. However, Gruneisen parameter measurements for concrete are typically around 0.1. For this reason, the equation-of-state ignores the Gruneisen parameter and Hugoniot pressure is used as total pressure. This, of course, means that internal energy changes have no effect on pressure, and the equation-of-state could not be used for energy deposition problems.

The variable, f'_c, is set to the CGS equivalent of 5,000 PSI. If other strengths are desired, it can be changed in the data statement that now sets it. Changing f'_c will vary the fracture point and the yield surface. It will have no effect on the pressure.

The sand equation-of-state was developed to simulate a dry 40 percent porosity quartz sand. It combines low pressure hydrostatic data from Reference 14 with pure quartz Hugoniot data from Van Thiel (Reference 10). The sand model assumes that strength is negligible at pressure levels of interest. This is a valid assumption as long as strength levels of a few hundred PSI are

unimportant in a calculation. There is evidence that sand can achieve such small levels of strength when highly crushed.

Sand has a very small Gruneisen parameter which can be ignored. The equation-of-state, then, is simply a pressure versus excess compression fit.

The virgin loading curve is described by a very small elastic segment:

$$P = K_o \mu$$

from $\mu = 0$ to $\mu = 0.0084$. K_o is computed from the initial density of 1.6 gm/cc and the initial sound speed of 0.61×10^5 cm/sec. The pressure level at the end of the elastic point is then 0.05 kilobar (approximately 50 atmospheres) the elastic segment is followed by two linear crushing regions:

$$P = 0.05 + 4.96\mu \qquad \text{(Kilobars)}$$

for μ from 0.0084 to 0.2

$$\text{and } P = 1.0 + 73.3\mu \qquad \text{(Kilobars)}$$

for μ from 0.2 to 0.425

At a μ of 0.425 and a pressure of 17.5 kilobars, the sand is assumed to have crushed out all voids and the loading curve follows the quartz Hugoniot:

$$P = 2.98\eta/(1-2.33\eta)^2 \qquad \text{(Kilobars)}$$

where:

$$\eta = 1 - \rho_o/\rho$$

To avoid the pole in this Hugoniot, the Hugoniot continues as a constant modulus curve at $\eta = 0.98/s$.

Unloading and reloading below the lockup point of 17.5 kilobars and above the elastic point of 0.05 kilobar follows a straight line curve which has a bulk modulus continuously varying from that at the lockup point to that along the elastic curve.

The sand is not allowed to sustain any tension.

Both concrete and sand require an extra zone variable, the maximum value of μ ever seen by the material. To successfully use these models, the user must set NHIST = 1 in the KEEL input deck. This will cause PLANK to increment NHIST by one which will allot one extra variable per zone after the variables required for carrying stresses. This will hold μ_{MAX} for concrete and/or sand.

3. Miscellaneous Solid/Liquid Equations-of-State

There are a few equations-of-state in HULL other than those following the Mie-Gruneisen formulation (the materials in Tables B-1 and B-2) and the concrete and sand models. These will now be briefly discussed.

Undetonated AFX-108 is very simply modelled for use in low velocity explosive-filled bomb and warhead impacts. The material name in HULL is AFX1. The Hugoniot pressure is calculated from:

$$P_H = B_1 \cdot \mu$$

where B_1 is set to 10.3×10^9 dynes/cm^2. The Gruneisen parameter is set to a constant value of 1.1 and pressure is computed from the Mie-Gruneisen equation with no controls or tensile values. The only control is a compressive one. μ is not allowed to exceed 0.9. This strange equation-of-state is the result of having only a single loading modulus for the material.

Tillotson equations-of-state exist in HULL for granite and tuff. They were developed for nuclear cratering calculations and should not be considered valid at low conventional weapons research pressure levels. They have not been exercised with FLUXER = 3, and the user is advised to avoid them if possible.

4. Air Equations of State

Two air equations-of-state are implemented in EOS 6. If the user simply asks for AIR, the equation-of-state uses the strange form:

$$P = (\gamma-1)\sqrt{\rho I \rho_o I_o}$$

where $\gamma = 1.4$ and I_o is set to return one atmosphere pressure at $\rho = \rho_o$. The purpose of this form is to provide air at approximately the correct density but with a very low sound speed. Small amounts of air are occasionally trapped in zones as the result of motion of more substantial media. High pressures in this trapped air can drive the time step to very small values and dominate the problem, when the pressure in the air, and perhaps even the presence of the air itself, is unimportant.

If correct air pressures are important, then the user must set AIREOS=1 as a SAIL option. AIREOS=1 insures that the DOAN-NICKEL variable gamma air model will be used. This is a highly accurate model over broad ranges of density and energy and is described in detail in Reference 26.

Because the code was used to simulate nuclear fireball calculations at various heights above sea level, HULL is capable of simulating several atmospheres (air initial conditions). These are described in Reference 27. If ATMOS is not specified, a constant, sea level atmosphere will be assumed.

5. High Explosive Equation of State

There are several high explosive equations of state used in HULL. The detonation products of standard military explosives are, for the most part, represented by the JWL formulation (Reference 18). In this formulation, pressure is computed from:

$$P = A\left(1 - \frac{\omega\bar{\eta}}{R_1}\right) e^{-\left(R_1/\bar{\eta}\right)} + B\left(1 - \frac{\omega\bar{\eta}}{R_2}\right) e^{-\left(R_2/\bar{\eta}\right)} + \omega\rho I$$

where $\bar{\eta} = \rho/\rho_o$, I is internal energy per unit mass and A, B, R_1, R_2 and ω are material constants.

This equation-of-state is implemented for Composition B, Octol (Reference 22) and C4 (Reference 23). JWL data exists for implementation for several more explosives (References 18-23).

The JWL equation has proven especially useful in accurately predicting metal acceleration. Its utility largely derives from the manner in which the

material constants are determined. They are forced to fit several consistency criteria such as total deliverable energy equal to that measured calorimetrically, pressure at Chapman-Jouget (CJ) density equal to measured CJ pressures, etc. The remaining parameters (R_1, R_2, and ω) are fit through an iterative experimental/calculational procedure involving measured velocities of cylinders and spheres.

At low densities, the last term in the JWL equation dominates. HULL takes advantage of the calculational simplification offered by this single term at high expansions. Whenever density of the detonation products decreases to one-tenth initial density, pressure is computed from the single $\rho\omega I$ term.

For densities greater than one-tenth ambient, the JWL pressure does not proceed to zero as energy proceeds to zero. This situation is avoided in HULL by decreasing pressure whenever energy is less than 1×10^9 ergs/gm. The calculated pressure is multiplied by $I/1 \times 10^9$. This arbitrary factor allows low energy, relatively high density gases to react more realistically. By the time energy has dropped from initial 4 to 5×10^{10} erg/gm levels to 1×10^9, the explosive has completely delivered its energy to surrounding media and is merely expanding to fill in the available grid regions.

The JWL equation-of-state calculates a temperature for editing purposes only from the equation:

$$T(^{o}K) = C_1 I + C_2$$

where C_1 is 1.65×10^{-8} and C_2 is 1375. These constants were found to fit available data for Composition B and are used for all JWL explosives.

The JWL constants A, B, R_1, R_2 and ω are designated as B_1, B_2, B_3, B_4, and B_5, respectively, in HULL. Values of these constants are listed in Table B-3.

HULL can provide burn progression (in 2D only) based on the CJ condition (D=C+U) and the direction of pressure gradients in the unburned explosive (BURN=1) or a programmed burn can be used (BURN=2). A programmed burn is suggested when an accurate detonation velocity is desired and when zone sizes are such that pressure at the burn front is significantly less than C_J pressure. Values of detonation velocity, D, are included in Table B-3 for use in

BURN=2. Pressure and $\gamma_{CJ} = C_p/C_v$ values at the CJ front are also included in the table for calculational comparison purposes.

If BURN=2 is desired, the user inserts values for TDET (time that the explosive region begins burning), VDET (detonation front velocity), and XDET, YDET, and ZDET (ignition point). These values are inserted as the last element in the PACKAGE setting up the explosive region.

TABLE B-3. JWL EXPLOSIVE CONSTANTS (CGS UNITS)

EXPLOSIVE	HULL NAME	ρ_0	I_0	A	B	R_1	R_2	ω
COMP B (64 RDX, 36 TNT)	CBBRN	1.72	4.92×10^{10}	5.242×10^{12}	0.07678×10^{12}	4.2	1.1	0.34
OCTOL	OCTBRN	1.82	5.27×10^{10}	7.486×10^{12}	0.1338×10^{12}	4.5	1.2	0.375
C4 (91 RDX, 5.3 DI(Z-ETHYLHEXYL), 2.1 POLYISOBUTYLENE, 1.6 MOTOR OIL).	C4BRN	1.601	5.62×10^{10}	6.0977×10^{12}	0.1295×10^{12}	4.5	1.4	0.25

EXPLOSIVE	NAME	D	C	γ_{cj}	P_{cj}
COMP B	CBBRN	7.98×10^5	0.01082×10^{12}	2.706	295×10^9
OCTOL	OCTBRN	8.48×10^5	0.01167×10^{12}	2.83	342×10^9
C4	C4 BRN	8.193×10^5	0.01043×10^{12}	2.838	280×10^9

It is sometimes convenient to calculate the work performed by an explosive in expanding from the CJ point to some density ρ.

The JWL adiabat (isentrope) is given by:

$$P = A\, e^{-(R_1/\bar{\eta})} + B\, e^{-(R_2/\bar{\eta})} + C(\bar{\eta})^{\omega+1}$$

the values of C for this adiabat are also given in the table. Integration of $-P\, d\rho/\rho^2$ from ρ_{CJ} to the density of interest will provide deliverable energy.

Other detonation product equations-of-state exist in HULL. They can be used (cautiously) for ANFO, BARATOL, TNT, PBX9404, and Pentolite. They use various forms for the equation-of-state and were developed for nuclear explosive simulation purposes.

ANFO, TNT, and Pentolite detonation products pressures were fit to the LSZK equation (References 24 and 25)

$$P = B_1 \rho I + B_2 \rho^{B_3}$$

with the parameter values as seen in Table B-3. The TNT and Pentolite data are considered to be very accurate at large expansions and were extensively tested. The equations have not been checked for near C_J behavior accuracy which would be dominant in metal acceleration problems. We suggest application of the JWL equation for this region. The ANFO data used in HULL was from a preliminary ANFO model developed by Lawrence Livermore Laboratory. Better ANFO data now exists and is contained in Reference 20. HULL has not been updated because of the lack of interest in ANFO in conventional munitions work.

The Baratol equation-of-state uses a gamma law formulation simply because better data does not seem to be available. The material parameters for this explosive are also given in Table B-3. Based on comparisons with other explosives, we can state that the constant gamma assumption will provide metal velocities close to 20 percent too high.

A PBX9404 fit to the JWL equation-of-state is included in HULL. The fit was further modified to reproduce airblast data in the range of 15 kilobars and below. It should not be used for metal acceleration.

The predetonation equations-of-state in HULL mostly use the form:

$$P = B_1\mu + B_2\mu^2 + B_3\mu^3 + \alpha\rho I$$

This provides a simple approximation to pressure for solids expected to load along a Hugoniot and for which subsequent unloading and reloading are unimportant. The pressure is not allowed to become negative in this equation-of-state.

Temperature is calculated for edit purposes only and is fit using the equation:

$$T = C_1 I + C_2 \quad \text{if } T > 288°K$$

$$T = C_3 I \quad \text{if } T < 288°K$$

Values of the coefficients used in the pressure and temperature calculations are included in Table B-4.

TABLE B-4. PREDETONATION EXPLOSIVE PARAMETERS (CGS UNITS)

EXPLOSIVE	HULL NAME	B_1	B_2	B_3	C_1	C_2	C_3
COMP B	COMP B	1.282×10^{11}	0	1.1932×10^{11}	3.36×10^{-8}	200	1.101×10^{-7}
OCTOL	OCTOL	1.282×10^{11}	0	1.1932×10^{11}	3.36×10^{-8}	200	1.101×10^{-7}
TNT	TNT	1.282×10^{11}	0	1.1932×10^{11}	3.312×10^{-8}	200	1.085×10^{-7}
PENTOLITE	PEN	1.319×10^{11}	3.614×10^{11}	2.295×10^{11}	3.312×10^{-8}	200	1.085×10^{-7}
ANFO	ANFO	2.4×10^{10}	1.08×10^{11}	7.2×10^{10}	3.3×10^{-8}	200	1.081×10^{-7}

The Baratol (HULL name BARTOL) predetonation equation is a Mie-Gruneisen form. The coefficients are:

$$C_o = 2.4 \times 10^5$$

$$s = 1.66$$

$$_o = 0.2$$

Temperature fits the form:

$$T = C_1 I + C_2$$

where $C_1 = 3.36 \times 10^{-8}$ and $C_2 = 200$.

6. High Pressure Geologic Equation-of-State

A high pressure equation-of-state for geologic materials was added to HULL to provide the necessary material properties for the calculation of meteor craters. It is designed primarily to provide a good fit to Hugoniot data between 50 kilobars and 1 megabar. Many details of the Hugoniot are only grossly approximated below 50 kb and so the equation-of-state should be used with caution in this area. It is based on the information in References 28 and 29 modified to allow a fit to a Mie-Gruneisen equation-of-state.

The Hugoniot is modelled in several connected sections. The Hugoniot lock-up curve is the curve which describes material behavior for a geologic material with no voids. The crush-up curve describes how a material with a non-zero void fraction behaves prior to reaching a zero-void condition. In this formation, ρ_c is the density at ambient conditions and ρ_r is the density at ambient pressure for the zero-void material. The crush-up loading curve is described by a single bulk modulus, K_1. The lock-up loading curve is described in several segments. A constant bulk modulus segment connects the 1-atmosphere point with the point at which the crush-up curve meets the lock-up curve (μ_L, P_L). This constant bulk modulus curve then turns into a variable modulus curve between P_L and P_{HI}. The curve is described by the Schuster-Isenberg form

$$P_H = K_m \mu_r - (K_m - K_o)\ \mu^*\ (1 - e^{-\mu/\mu^*})$$

where $\mu_r = \rho/\rho_r - 1$ and K_o and K_m are initial and maximum bulk moduli and μ^* is a parameter which determines the density range between the points at which K_o and K_m dominate. At the point (μ_{HI}, P_{HI}) the Hugoniot shifts again to a constant modulus form

$$P_H = K_{HI} (\mu_r - \mu_{HI}) + P_{HI}$$

the unloading path taken depends upon μ_{max}, which is the maximum value of μ seen by the material. If μ_{max} is greater than μ_L, then all unloading and reloading occurs along the single lock-up curve. If μ_{max} is less than μ_L, then unloading and reloading occur along a path parallel to the lock-up curve. The Hugoniot pressure is unlimited in tensile extent, but the total pressure is limited to values greater than P_{min}, where P_{min} is a material parameter. In addition, total tensile pressure is limited as specific internal energy, I, approaches I_m, the value at which strength effects are no longer important. In this case the total tensile pressure is multiplied by

$$2.5 \text{ Max } (I_m - \text{Max } (I, 0.6I_m),0)/I_m$$

This formulation insures that tensile pressure values which have passed the P_{min} test are not modified unless I exceeds 0.6 I_m.

The total pressure is calculated using the Mie-Gruneisen form

$$P = P_H (1 - \Gamma\mu/2) + \Gamma\rho I + P_o$$

where Γ, the Gruneisen parameter, is given by

$$\Gamma = \Gamma_o\rho_o/\rho$$

subject to the restriction that Γ cannot exceed 2 Γ_o. P_o is given by

$$P_o = \frac{1.013\text{x}10^6}{Io} \text{ min } (I, Io) \text{ dynes/cm}^2$$

where Io is ambient (1 atmosphere) specific internal energy. P_o then insures that material pressure will initially be 1 atmostphere. The total solid or

liquid pressure. P, is limited by P_{MIN} and an I_m factor as discussed previously. In addition, it is limited to very small positive values if μ is less than -0.25 (the theory being that it is essentially rubble unable to support tension at these low densities).

In addition to P, a gas pressure, P_G, is calculated if I exceeds I_m. This pressure is calculated using a gamma law formulation

$$P_G = (\gamma-1)\ \rho\ (I-I_m)$$

P_G is compared to the solid/liquid pressure, P, and P is reset to the maximum of the two values.

The shear modulus, G, is calculated from

$$G = (G_m\mu - (G_m-G_o)(1-e^{\mu/\mu_G}))(1-I/I_m) \qquad \text{if } I < I_m$$

and

$$G = 0 \qquad \text{if } I \geq I_m$$

where G_o, G_m, and μ_G are material parameters.

The yield strength, Y, is calculated from

$$Y = Y_r \cdot Y_I \cdot \text{Min}\ (A + BP,\ Y_{max})$$

where Y_r is a rubble factor set as follows

$$Y_r = 1 \qquad \text{if } \mu > -0.2$$

$$Y_r = 0 \qquad \text{if } \mu < -0.25$$

$$Y_r = 20\ (\mu + 0.25) \qquad \text{if } -0.25 < \mu < -0.2$$

The factor insures that rubble cannot sustain any yield strength. Y_I is an internal energy factor and is given by

$$Y_I = 0 \qquad \text{if } I \geq I_m$$

$$Y_I = 1 - I/I_m \qquad \text{if } I < I_m$$

This is a thermal softening factor. The term A + BP provides a Mohr-Coulomb yield strength which reaches saturation at $Y = Y_{max}$.

Temperature is calculated for edit purposes by arbitrarily setting ambient internal energy to 1 x 10^8 ergs/gm and computing a C_v value to return 288 K at ambient energy.

The attached Table B-5 lists materials which utilize this equation-of-state, the HULL names for the materials, and the material quantities built into HULL for the materials.

TABLE B-5. HIGH PRESSURE GEOLOGIC PROPERTIES

MATERIAL	Dry Moenkopi Sandstone	Dry Kaibab Limestone	Dry Coconino Sandstone	Saturated Coconino Sandstone	Limestone
HULL NAME	MOENK	KAIBAB	DRYCOC	WETCOC	LIMEST
ρ_o (gm/cc)	2.25	2.30	2.03	2.35	2.58
ρ_r (gm/cc)	2.68	2.70	2.68	2.35	2.70
K_1 (dynes/cm²)	60.E9	100.E9	60.E9	225.E9	500.E9
K_o (dynes/cm²)	450.E9	600.E9	450.E9	225.E9	600.E9
K_m (dynes/cm²)	700.E9	800.E9	700.E9	580.E9	800.E9
K_{HI} (dynes/cm²)	4.656E12	2.942E12	4.656E12	2.22E12	2.25E12
μ_{Lock}	0.029208	0.03541	0.0488	0	0.196
μ_{HI}	0.65	0.2963	0.75373	0.74468	0.33333
μ^*	0.25	0.4	0.25	0.3	0.4
μ_G	0.0015	1.0	0.0015	0.0015	1.0
P_{min} (dynes/cm²)	-0.667E8	-0.15E9	-0.667E8	-0.667E8	-0.15E9
P_{Lock} (dynes/cm²)	13.55E9	22.86E9	23.08E9	0	130.87E9
P_{HI} (dynes/cm²)	397.14E9	195.18E9	468.17E9	334.31E9	221.44E9
G_o (dynes/cm²)	50.E9	150.E9	50.E9	45.E9	150.E9
G_m (dynes/cm²)	120.E9	150.E9	120.E9	110.E9	150.E9
E_m (ergs/gm)	37.E9	10.E9	37.E9	25.E9	10.E9
A (dynes/cm²)	5.E7	0.2E9	5.E7	5.E7	0.2E9
B	0.75	1.0	0.75	0.75	1.0
Y_{max} (dynes/cm²)	3.E9	3.E9	3.E9	3.E9	3.E9
Y_o	0.5	0.5	0.5	0.5	0.5
γ-1	0.4	0.4	0.4	0.4	0.4
$I_{ambient}$ (ergs/gm)	1.E8	1.E8	1.E8	1.E8	1.E8

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